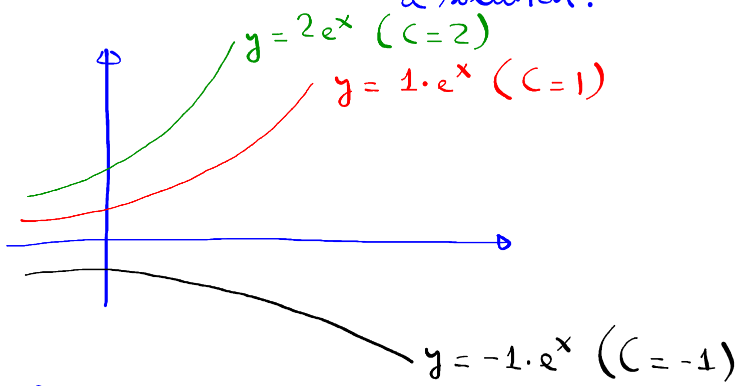
Family of Solutions

E.g. $y' = y$

$y = C \cdot e^x$ where C is any constant will be a solution.



This is an example of a 1-parameter family of solutions to the equation.

E.g. 4. $y = c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2$.

3-parameter family of solutions to the ODE:

$$x^3 \boxed{y'''} + 2x^2 \boxed{y''} - x\boxed{y'} + y = 12x^2.$$

$$y' = -c_1 x^{-2} + c_2 + c_3 \ln x + c_3 + 8x$$

$$y'' = \boxed{2c_1 x^{-3} + \frac{c_3}{x} + 8}$$

$$y''' = \boxed{-6c_1 x^{-4} - c_3 x^{-2}}$$

Plug in:

$$\cancel{-6c_1 x^{-1}} - \cancel{c_3 x} + \cancel{4c_1 x^{-1}} + 2\cancel{c_3 x} + 16x^2$$

$$\cancel{+c_1 x^{-1}} - \cancel{x c_2} - \cancel{c_3 x \ln x} - \cancel{c_3 x} - 8x^2$$

$$\cancel{+c_1 x^{-1}} + \cancel{c_2 x} + \cancel{c_3 x \ln x} + 4x^2 = \boxed{12x^2}$$

A particular solution

$y' = y$

$y = Ce^x$ is a 1-para. family of solutions to this D.E.

Find y such that : An I. V. P

$$\underbrace{y' = y \quad \text{and} \quad y(0) = 2019}$$

Solution : $y = 2019e^x$ → particular solution

E.g. 5. $y = c_1 \cos(x) + c_2 \sin(x)$

$$y'' + y = 0 \ ; \ y\left(\frac{\pi}{2}\right) = 0 \ ; \ y'\left(\frac{\pi}{2}\right) = 1$$

$$y\left(\frac{\pi}{2}\right) = c_1 \cos\left(\frac{\pi}{2}\right) + c_2 \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = 0$$

0

$$\boxed{c_2 = 0}$$

$$y' = -c_1 \sin(x) + c_2 \cos(x)$$

$$y'\left(\frac{\pi}{2}\right) = -c_1 \sin\left(\frac{\pi}{2}\right) + c_2 \cos\left(\frac{\pi}{2}\right) = 1$$

$$-c_1 = 1 \longrightarrow \boxed{c_1 = -1}$$

Particular solution to this IVP: $\boxed{y = -\cos x}$

E.g. 6

$$y = c_1 e^{x} + c_2 e^{-x}$$

$$y' = c_1 e^{x} - c_2 e^{-x}$$

$$y'' = c_1 e^{x} + c_2 e^{-x} = y.$$

The D.E. is $\boxed{y'' = y}$