this gives us:

$$g'(y) = 12y^2$$

$$\longrightarrow g(y) = 4y^3.$$

So, $F(x,y) = x^3y + 4x^2y^2 + 4y^3$

Conclusion: solution to the given equation will be the 1-para. family:

$$x^3y + 4x^2y^2 + 4y^3 = C$$

C is any constant.

E.g. 3

① Check for exactness

$$\frac{\partial M}{\partial y} = 2y\cos x - 3x^2$$

$$\frac{\partial N}{\partial x} = 2y\cos x - 3x^2$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Hence, exact

② Integrate M w.r.t. x

$$F(x,y) = \int M\,dx + g(y)$$

$$F(x,y) = \int (y^2\cos x - 3x^2y - 2x)\,dx + g(y)$$

$$= y^2 \sin x - x^3 y - x^2 + g(y)$$

3 Take partial of F w.r.t. y and set it equal to N to solve for g:

$$\frac{\partial F}{\partial y} = 2y\sin x - x^3 + g'(y)$$

Set equal to N: $\cancel{2y\sin x} \cancel{- x^3} + g'(y) =$

$$\underbrace{\cancel{2y\sin x} \cancel{- x^3} + \ln y}_{N}$$

So, $g'(y) = \ln y$

Integration by parts:

$$\longrightarrow g(y) = \int \ln y \, dy \qquad \begin{cases} u = \ln y \to du = \frac{1}{y} dy \\ dv = dy \to v = y \end{cases}$$

$$= y \ln y - \int dy = y \ln y - y.$$

So, $g(y) = y \ln y - y$.

So, $F(x,y) = y^2 \sin x - x^3 y - x^2 + y \ln y - y$.

1-para family of sol:

$$y^2 \sin x - x^3 y - x^2 + y \ln y - y = C$$