So $y^{-3}\dfrac{dy}{dx} = -\dfrac{1}{2}\dfrac{du}{dx}$.

Substitute everything back to the equation in Step 1

$$-\frac{1}{2}\frac{du}{dx} - u = e^{2x}$$

$\longrightarrow$ this is a first order linear equation.

③ Solve for u : $\longrightarrow$ $\underline{P}(x)$

$$\frac{du}{dx} + \boxed{2}u = -2e^{2x}$$

Integrating factor: $e^{\int 2\,dx} = e^{2x}$

$$\underbrace{e^{2x} \cdot \frac{du}{dx} + 2e^{2x} u}_{} = -2e^{2x} \cdot e^{2x}$$

$$\int \cancel{\frac{d}{dx}} \left[e^{2x} \cdot u \right] = \int -2e^{4x}\, dx$$

$$e^{2x} \cdot u = -2 \cdot \frac{e^{4x}}{4} + C$$

$$e^{2x} \cdot u = -\frac{1}{2} e^{4x} + C$$

$$u = -\frac{1}{2} e^{2x} + Ce^{-2x}$$

So, $\boxed{y^{-2} = -\frac{1}{2} e^{2x} + Ce^{-2x}}$

E.g. 4.

$$x^2 \frac{dy}{dx} - 2xy = 3y^4 \quad ; \quad y(1) = \frac{1}{2}.$$

$P(x)$ $Q(x)$

$$\frac{dy}{dx} - \frac{2}{x} y = \frac{3}{x^2} y^4 \rightarrow y^n$$

→ This is Bernoulli.

(1) Divide both sides by y^4

$$y^{-4}\frac{dy}{dx} - \frac{2}{x} y^{-3} = \frac{3}{x^2}$$

So, $y^{-4}\frac{dy}{dx} = -\frac{1}{3}\frac{du}{dx}$

(2) Make a substitution:

$$\boxed{u = y^{-3}} \longrightarrow \frac{du}{dx} = -3y^{-4}\frac{dy}{dx}$$

$$-\frac{1}{3}\frac{du}{dx} - \frac{2}{x}u = \frac{3}{x^2}$$ Multiply by -3

$$\frac{du}{dx} + \frac{6}{x}u = -\frac{9}{x^2}$$

Int. Factor: $I(x) = e^{\int \frac{6}{x}dx} = e^{6\ln x} = x^6$

Multiply both sides by int. factor:

$$\underbrace{x^6\frac{du}{dx} + 6x^5u} = -9x^4$$

$$\int \frac{d}{dx}\left[x^6u\right] = -\int 9x^4$$

$$x^6 u = -\frac{9x^5}{5} + C$$

$$u = -\frac{9}{5}x^{-1} + Cx^{-6}$$

$$\boxed{y^{-3} = -\frac{9}{5}x^{-1} + Cx^{-6}}$$