Lecture 7 - Preliminary Theory about linear DEs of order n.

Goal: Discuss some general results on the nature of the solutions to these equations.

The form of a linear DE of order n:

$$\boxed{a_n(x)\cdot\frac{d^n y}{dx^n} + a_{n-1}(x)\cdot\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)}$$

* Case: $g(x) = 0$

$$a_n(x) \cdot \frac{d^n y}{dx^n} + \cdots + a_1(x) \frac{dy}{dx} + a_0(x) = 0$$

$\longrightarrow$ Homogeneous linear equation of order n.

* Case: $g(x) \neq 0$

$\longrightarrow$ Non Homogeneous linear equation of order n

* Case: Solve the equation subject to:

$$\boxed{y(x_0) = y_0 \ ; \ y'(x_0) = y_1 \ ; \ \cdots, \ y^{(n-1)}(x_0) = y_{n-1}}$$

Initial conditions.

Existence and Uniqueness Theorem for an nth - order I.V.P.

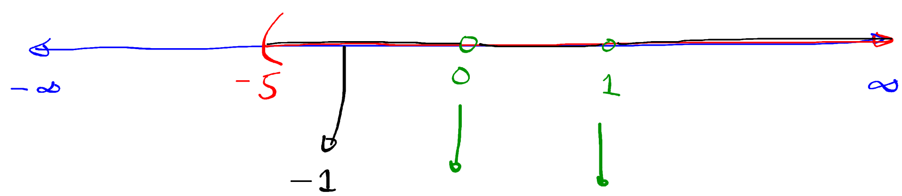
E.g. 1: $\boxed{x(x-1)y''' - 3x y'' + 6x^2 y' - (\cos x) y = \sqrt{x+5}}$

3^{rd} order, linear.

Initial conditions: $y(x_0) = 2$; $y'(x_0) = 6$; $y''(x_0) = 7$

(1) $x_0 = -1 \longrightarrow \boxed{y(-1) = 2;\ y'(-1) = 6;\ y''(-1) = 7}$

Largest interval on which all coeff. functions and RHS is continuous is $(-5, \infty)$



leading coeff. $= 0$

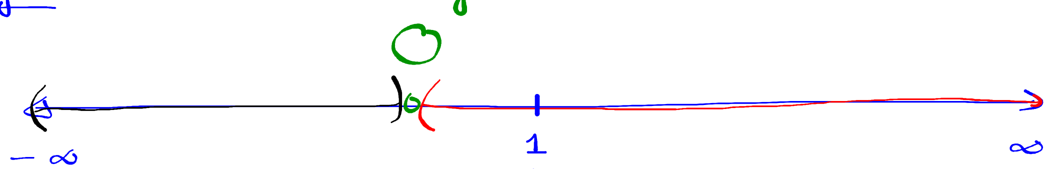
When $x_0 = -1$, the largest interval for which the theorem apply is: $(-5, 0)$

when $x_0 = \frac{1}{3}$, the largest interval $= (0, 1)$

when $x_0 = 10$, ______________ $(1, \infty)$

E.g. 2

leading $= 0$



initial condition

(a) $I = (0, \infty)$: largest interval for which the I.V.P has a unique solution.

(b) $y = c_1 + c_2 x + c_3 x^3 + x^2$

$y(1) = 2$; $y'(1) = -1$; $y''(1) = -4$

$2 = c_1 + c_2 + c_3 + 1 \rightarrow \boxed{c_1 + c_2 + c_3 = 1}$

$y' = c_2 + 3c_3 x^2 + 2x \rightarrow -1 = c_2 + 3c_3 + 2 \rightarrow \boxed{c_2 + 3c_3 = -3}$

$y'' = 6c_3 x + 2 \longrightarrow -4 = 6c_3 + 2$

$\longrightarrow \boxed{6c_3 = -6}$

$\longrightarrow \boxed{c_3 = -1}$

$\boxed{c_2 = 0}$

$\boxed{c_1 = 2}$

$$\boxed{y = 2 - x^3 + x^2}$$

$\longrightarrow$ the unique solution to the I. V. P.