→ Complementary function: $y_c = c_1 e^{x/2} + c_2 x e^{x/2}$

Theorem says that the solution to the non homo. equation:

$$y = y_c + y_p$$

any particular solution to the non. homo. eqn.

In this case, $\boxed{y_p = -3\sin x + 4\cos x}$.

So, the general sol. to non homo. eqn:

$$y = c_1 e^{x/2} + c_2 x e^{x/2} + (-3\sin x + 4\cos x)$$

Verify that $\boxed{y_p = -3\sin x + 4\cos x}$ is in fact a particular sol to

$$4y'' - 4y' + y = 25\sin x$$

$$y' = -3\cos x - 4\sin x$$

$$y'' = 3\sin x - 4\cos x$$

$$4(3\sin x - 4\cos x) - 4(-3\cos x - 4\sin x) + (-3\sin x + 4\cos x)$$

$$= 25\sin x = \text{RHS.} \checkmark$$

E.g. 6.

$L_1 = D^2 + D - 2. \qquad y = x^3 - 1$

$$L_1 y = (D^2 + D - 2)(x^3 - 1)$$

$$= \underbrace{D^2(x^3 - 1)} + \underbrace{D(x^3 - 1)} - 2(x^3 - 1)$$

$$= 6x + 3x^2 - 2x^3 + 2$$

$$L_1 y = \boxed{-2x^3 + 3x^2 + 6x + 2}$$

$$L_2 = x^3 D^3 + x^2 D^2 - xD + 6$$

$$L_2 y = x^3 \cdot (6) + x^2 \cdot (6x) - x \cdot (3x^2) + 6(x^3 - 1)$$

$$= 6x^3 + 6x^3 - 3x^3 + 6x^3 - 6$$

$$L_2 y = 15x^3 - 6$$

E.g. 7:

$$\left.\begin{array}{l} D^2+3D+2 \\ (D+2)(D+1) \end{array}\right\}$$ Show that these are the same operator

$$(D^2+3D+2)\,y = y'' + 3y' + 2y$$

$$(D+2)\underbrace{(D+1)\,y}_{} = (D+2)(\underbrace{y'+y}_{})$$

$$= D(y'+y) + 2(y'+y)$$

$$= y'' + y' + 2y' + 2y = y'' + 3y' + 2y$$