E.g. 1.

① $y'' - 3y' + 2y = 0$

Char. Eqn: $\boxed{m^2 - 3m + 2 = 0}$

Solve: $(m-1)(m-2) = 0$

2 real, distinct solutions to the char. eqn:

$m_1 = 1 \; ; \; m_2 = 2$

General sol to the DE:

$$\boxed{y = c_1 e^{x} + c_2 e^{2x}}$$

② $y'' + 8y' + 16y = 0$

Char Eqn: $m^2 + 8m + 16 = 0$

Solve Char Eqn: $(m+4)^2 = 0$

Sol. to char Eqn: $m_1 = m_2 = -4$

→ 1 real solution

→ General solution to D.E.

$$y = c_1 e^{-4x} + c_2 x e^{-4x}$$

③ $y'' - 4y' + 7y = 0$

Char Eqn: $m^2 - 4m + 7 = 0$.

Solve char eqn: $m_1 = \frac{4 + \sqrt{16 - 28}}{2}$; $m_2 = \frac{4 - \sqrt{16-28}}{2}$

$(\sqrt{-1} = i)$ $m_1 = \frac{4 + \sqrt{-12}}{2}$; $m_2 = 4 - \sqrt{-12}$

$m_1 = \frac{4 + i\sqrt{12}}{2}$; $m_2 = \frac{4 - i\sqrt{12}}{2}$

$m_1 = \frac{4 + 2i\sqrt{3}}{2}$; $m_2 = \frac{4 - 2i\sqrt{3}}{2}$

$m_1 = \underset{\alpha}{2} + i\underset{\beta}{\sqrt{3}}$; $m_2 = 2 - i\sqrt{3}$

(2 complex (non real) solutions appearing as a pair of conjugate)

General sol. to DE:

$$y = e^{2x}\left(c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)\right)$$

E.g.2: $y = e^{-x}\left(c_1 \cos(4x) + c_2 \sin(4x)\right)$

$m^2 + 2m + 17 = 0$ (Char. Eqn)

$m^2 + 2m = -17$

$m^2 + 2m + 1 = -16$

$(m+1)^2 = -16 \longrightarrow m + 1 = \pm\sqrt{-16} = \pm 4i$

$m = -1 \pm 4i$

$$m_1 = \frac{-2 + \sqrt{4 - 68}}{2} \quad ; \quad m_2 = \frac{-2 - \sqrt{4 - 68}}{2}$$

$$m_1 = \frac{-2 + \sqrt{-64}}{2} \quad ; \quad m_2 = \frac{-2 - \sqrt{-64}}{2}$$

$$m_1 = \frac{-2 + 8i}{2} \quad ; \quad m_2 = \frac{-2 - 8i}{2}$$

$$m_1 = -1 + 4i \quad ; \quad m_2 = -1 - 4i$$

General sol: $y = e^{-x}\left(c_1 \cos(4x) + c_2 \sin(4x)\right)$

$y(0) = 1$: $\boxed{1 = c_1}$

$y'(0) = -1$

$$y = e^{-x}\left(c_1 \cos(4x) + c_2 \sin(4x)\right)$$

$$y' = -e^{-x}\cdot\left(c_1 \cos(4x) + c_2 \sin(4x)\right) + e^{-x}\cdot\left(-4c_1 \sin(4x) + 4c_2 \cos(4x)\right)$$

$y'(0) = -1$:

$$-1 = -(c_1) + 1\cdot(4c_2)$$

$$-1 = -c_1 + 4c_2$$

Since $c_1 = 1 \longrightarrow -1 = -1 + 4c_2 \rightarrow 4c_2 = 0 \rightarrow c_2 = 0.$

Final sol to IVP: $\boxed{y = e^{-x}\cos(4x)}$

Higher order linear Homogeneous equation:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

CharEqn:

$$a_n m^n + a_{n-1} m^{n-1} + \cdots + a_1 m + a_0 = 0$$

this is an algebraic equation in the variable m with degree n.

When we solve this char. eqn, we could run into a few major cases