Case 1: All roots are real and distinct.

E.g. Char Eqn: $(m-2)(m-3)(m+4)(m+5)=0$

Sol to char eqn: $m=2$; $m=3$; $m=-4$; $m=-5$

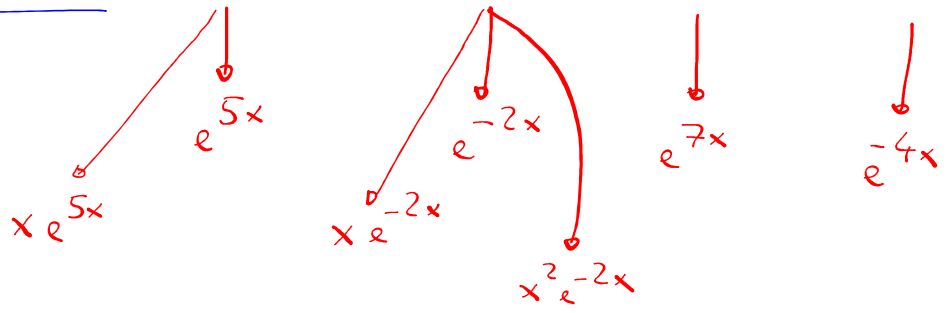
General sol to D.E.:

$$y = c_1 e^{2x} + c_2 e^{3x} + c_3 e^{-4x} + c_4 e^{-5x}$$

Case 2: All roots are real, some are repeated.

E.g. $(m-5)^2(m+2)^3(m-7)(m+4)=0$ (Char Eqn)

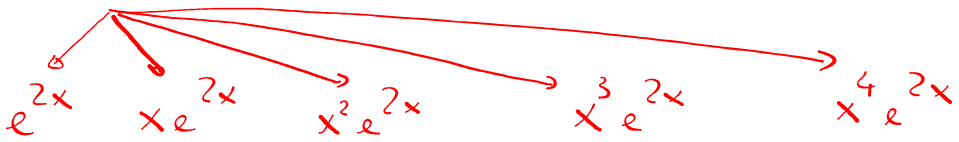
Sol to char: $m=5$; $m=-2$; $m=7$; $m=-4$



General to the D.E.

$$y = c_1 e^{5x} + c_2 x e^{5x} + c_3 e^{-2x} + c_4 x e^{-2x} + c_5 x^2 e^{-2x} + c_6 e^{7x} + c_7 e^{-4x}.$$

E.g. $(m-2)^5$ (char eq)



Case 3: All roots are complex (non real)

Note: complex solutions always appear in conjugate pairs

E.g. Suppose that the char eqn has the following roots:

$2+3i$; $2-3i$; $1+i\sqrt{2}$, $1-i\sqrt{2}$, $5i$, $-5i$

$e^{2x}\cos(3x)$; $e^{2x}\sin(3x)$; $e^{x}\cos(\sqrt{2}x)$, $e^{x}\sin(\sqrt{2}x)$

$\cos(5x)$, $\sin(5x)$

General solution: → combine these.

Case 4: Repeated complex roots

The char eqn has the following roots:

$2 - 3i$, $2 + 3i$

i repeated 3 times; repeated 3 times.

$e^{2x}\cos(3x)$; $e^{2x}\sin(3x)$

$x \cdot e^{2x}\cos(3x)$; $x \cdot e^{2x}\sin(3x)$

$x^2 \cdot e^{2x}\cos(3x)$; $x^2 \cdot e^{2x}\sin(3x)$

} 6 L.I. solutions

→ General sol: combine these with constants.

E.g. 3:

Char Eqn: $m^3 + 3m^2 - 4m - 12 = 0$

$$m^2(m+3) - 4(m+3) = 0$$

$$(m+3)(m^2-4) = 0$$

$$(m+3)(m-2)(m+2) = 0$$

$m = -3$; $m = 2$; $m = -2$

General Sol to the D.E.

$$y = C_1 e^{-2x} + C_2 e^{2x} + C_3 e^{-3x}$$

E.g.4. $y^{(4)} - 8y'' + 16y = 0$ $\quad u^2 - 8u + 16$, $(u-4)^2$

Char Eqn: $m^4 - 8(m^2)^u + 16 = 0$

$$(m^2 - 4)^2 = 0$$

$$[(m+2)(m-2)]^2 = 0$$

$$(m+2)^2 (m-2)^2 = 0$$

$m = -2$; $m = 2$

repeated twice repeated twice

e^{-2x} $\quad x e^{-2x}$ $\quad e^{2x}$ $\quad x e^{2x}$

General sol to D.E.:

$$y = c_1 e^{-2x} + c_2 x e^{-2x} + c_3 e^{2x} + c_4 x e^{2x}$$