# Definitions and Terminology

# Recommended reading from Zill's DEs with BVP-7e: Section 1.1 (pg 2-9): Examples 1 through 4. Section 1.2 (pg 13-17): Examples 1 through 3.

#### **Ordinary Differential Equations - Definitions**

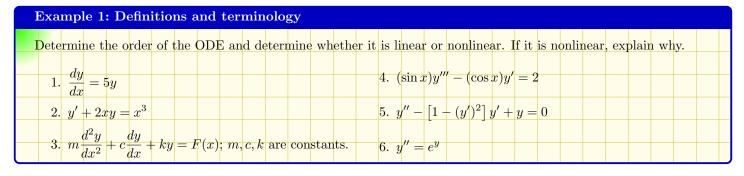
Let y = f(x) be a function of x. An ordinary differential equation (ODE) is an equation involving x, the function y = f(x) and one or more of its derivatives. The order of an ODE is the order of the highest derivative in the equation.

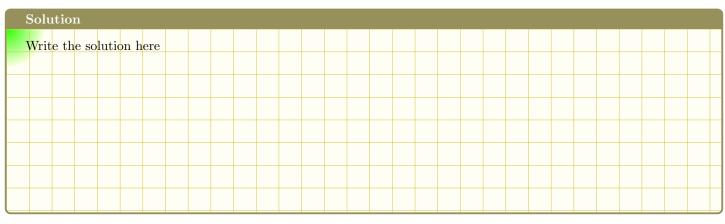
A linear ODE is an equation of the form

 $a_0(x)y + a_1(x)y' + a_2(x)y'' + \dots + a_n(x)y^{(n)}(x) = g(x).$ 

In a linear ODE, the power of each term involving y and its derivatives  $y', y'', \ldots$  is at most 1 and the coefficients  $a_0, a_1, \ldots, a_n$  either are constant or depend only on x.

A nonlinear ODE is an ODE that is not linear. Nonlinear functions of the dependent variable y or its derivative such as  $y^2$ ,  $\cos(y)$ ,  $e^y$ ,  $\ln(y)$ ,  $\arctan(y')$ ,  $(y')^3$  cannot appear in a linear ODE.





#### Solution of an ODE - Explicit vs. Implicit

The function y = f(x) defined on an interval I is an **explicit solution** of an ODE if replacing y by f(x), y' by  $f'(x), \ldots, y^{(n)}$  by  $f^{(n)}(x)$  reduces the equation to an identity.

A relation G(x, y) = 0 is an **implicit solution** of an ODE on an interval I = (a, b) if

- 1. The relation defines y as an implicit function of x on I, i.e., there exists a function  $y = \phi(x)$  that satisfies the relation and
- 2.  $y = \phi(x)$  satisfies the ODE on *I*.

E	xai	mp	ole	2:	E	xpli	$\mathbf{cit}$	so	olu	tio	n																					
Ve	erif	y t	ha	t t.	he	indi	cate	ed	fur	ncti	on i	s an	exp	licit	solı	itior	ı of	$_{\mathrm{the}}$	give	n O	DE.											
																		~	,,						,							
	1.	(1	+	$x^2$	)y'	= x	y;		<i>y</i> =	= V	1+	$x^2$						2. y	<i>ľ</i> +	y =	tan	x;	y	= -	$-(\cos$	(x)	n(se	$ec x \cdot$	+ ta	nx)		

	Solu	itio	n															
,	Writ	e th	e so	lutio	on h	$\mathbf{ere}$												

Example 3: Implicit solution	
Determine whether the relation $G(x,y) = x^2 + y^2 - 25 = 0$ is an implicit solution of the $G(x,y) = x^2 + y^2 - 25 = 0$	DDE
yy' + x = 0 on the interval $I = (-5, 5)$ .	

9	Solu	itio	n															
	Writ	e th	e so	lutio	on h	$\operatorname{ere}$												

## Family of solutions - general solution - particular solution - initial conditions

A solution containing a constant c is called a **one-parameter family of solutions**. A solution containing n contants  $c_1, c_2, \ldots, c_n$  is called a **n-parameter family of solutions**. The constants  $c_1, \ldots, c_n$  are the parameters. A solution that does not contain parameters (constants) is called a **particular solution**.

If every solution of an ODE can be obtained from an *n*-parameter family of solutions by appropriate choices of the parameters, then the family is called the **general solution** of the ODE.

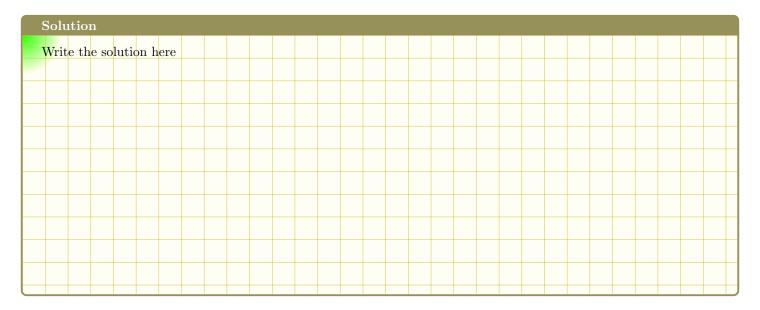
An initial value problem (IVP) is a problem in which we find a solution y(x) of an ODE so that y(x) also satisfies certain prescribed conditions, i.e., conditions that are imposed on the values of the unknown function y(x) and its derivatives at a specific number  $x_0$ . These conditions are called the **initial conditions** of the ODE and for an order nth ODE are of the form

$$y(x_0) = y_0, y'(x_0) = y_1, \dots, y^{(n-1)}(x_0) = y_{n-1},$$

where  $y_0, y_1, \ldots, y_{n-1}$  are constants.

## Example 4: Family of solutions

Verif	v th	nat t	he :	3-pai	rame	eter	farr	ilv (	of fu	ncti	ons	u =	$c_1 x$	-1	$+ c_2$	r +	$c_3 x$	n x	+4x	$^2$ is	a fai	milv	of	solı	itior	ıs to	the	ODE	
	5			- P -				5														5							
										x	y'''	+2	$x^2y'$	· — :	<i>xy</i> ' -	- y :	= 12	2x2.											



#### Example 5: Particular solution

Given that  $y = c_1 \cos(x) + c_2 \sin(x)$  is a 2-parameter family of solutions of the ODE y'' + y = 0. Find the particular solution of the ODE that satisfies the initial conditions  $y(\pi/2) = 0$  and  $y'(\pi/2) = 1$ .

Solution									
Write the solution here									

Ex	car	npl	le 6	: Fi	nd	a di	iffer	$\mathbf{ent}$	ial o	equ	atio	n												
Fir	nd	a d	iffer	enti	al e	quat	ion	whe	se 2	-pai	ame	eter	fam	ily o	of so	luti	ons	is						
															x		-x							
													y	$= c_1$	[ <i>e</i> " -	$+c_2$	e ".							

Write the solution here I	