Special types of first order equations - Separable Equations

Recommended reading from Zill's DEs with BVP-7e: Section 2.2 (pg 44-50): Examples 1 through 4.

Solve Separable Differential Equations

A first order equation of the form
$$\frac{dy}{dx} = g(x)h(y)$$
is said to be **separable**.
Method of solution of a first order separable equation:

1. Rewrite the equation as $\dfrac{1}{h(y)}dy = g(x)dx$.
2. Integrate both sides $\displaystyle\int \frac{1}{h(y)}dy = \int g(x)dx$
3. Obtain a 1-parameter family of solutions: $H(y) = G(x) + C$ where $H(y)$ is an antiderivative of $\frac{1}{h(y)}$ and $G(x)$ is an antiderivative of $g(x)$.

To solve a initial value problem where the ODE is separable, we find the 1-parameter family of solutions and use the initial conditions to determine the parameter C.

Example 1: Solve a separable equation

Solve the linear, first order, separable equation $\dfrac{dy}{dx} = 3x^2y$

Solution

Write the solution here

Example 2: Solve an initial value problem

Solve the nonlinear, first order, initial value problem

$$\frac{1}{x}\frac{dy}{dx} = \frac{y\sin x}{y^2+1}, y(\pi) = 1.$$

Is your solution an implicit or an explicit solution?

Solution

Write the solution here

Example 3: Losing solutions

Find a 1-parameter family of solutions of the nonlinear, first order, separable equation

$$(y^2-1)dx - (2y+xy)dy = 0.$$

Determine two particular solutions which are not members of the family of solutions.

Solution

Write the solution here

Example 4: An Application - Newton's Law of Cooling

Newton's Law of cooling says that the rate of change of the temperature T of an object is proportional to the difference in the temperature of the surrounding environment and of the object. This gives the differential equation

$$\frac{dT}{dt} = k(S - T),$$

where T is the temperature of the object which is a function of time t, S is the constant temperature of the surrounding environment and k is a proportional constant.

1. Solve the equation for T. (Note that k and S are constants.)
2. Suppose a corpse was discovered in a room at midnight and its temperature was $80°$ F. The temperature of the room is constant at $65°$ F. Two hours later the temperature of the corpse dropped to $70°$ F. Determine the time of death. Assume the temperature of a corpse at time of death is $98.6°$ F.

Solution

Write the solution here