Wednesday, August 7, 2019 9:49 AM

HW9 #6.1

$$y''' - 6y'' = 3 - \cos(x)$$

Step 1: Homogeneous Eqn: $y''' - 6y'' = 0$

Char. Eqn: $m^3 - 6m^2 = 0 \longrightarrow m^2(m-6) = 0$

$\longrightarrow m = 0$ (repeated twice) ; $m = 6$.

$$y_c = c_1 + c_2 x + c_3 e^{6x}$$

Step 2: Find y_p

Form of y_p: $y_p = Ax^2 + B\cos x + C\sin x$

$y_p' = 2Ax - B\sin x + C\cos x$; $y_p'' = B\sin x - C\cos x$

$y_p''' = 2A - B\cos x - C\sin x$;

$$B\sin x - C\cos x - 6(2A - B\cos x - C\sin x) = 3 - \cos x$$

$$-12A + (-C + 6B)\cos x + (B + 6C)\sin x = 3 - \cos x$$

$(B + 6C) = 0$

$-12A = 3$; $B + 6C = 0$

$-C + 6B = -1$;

$\longrightarrow \begin{cases} B + 6C = 0 \\ 6B - C = -1 \end{cases}$; $B = -\frac{6}{37}$, $C = \frac{1}{37}$

$A = -\frac{1}{4}$

$$y_p = -\frac{1}{4}x^2 - \frac{6}{37}\cos x + \frac{1}{37}\sin x$$

Final solution: $\boxed{y = y_c + y_p = c_1 + c_2 x + c_3 e^{6x} - \frac{1}{4}x^2 - \frac{6}{37}\cos x + \frac{1}{37}\sin x}$

Practice Test #6

$$y''' + 2y'' - y' - 2y = \sin(3x) \; ; \; y(0) = 0 \; ; \; y'(0) = 0 \; ; \; y''(0) = 1$$

$$\mathcal{L}\{y'''\} + 2\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = \mathcal{L}\{\sin(3x)\}$$

$$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + 2\left[s^2 Y(s) - s y(0) - y'(0)\right] - \left[sY(s) - y(0)\right] - 2Y(s) = \frac{3}{s^2+9}$$

$$s^3 Y(s) - 1 + 2s^2 Y(s) - sY(s) - 2Y(s) = \frac{3}{s^2+9}$$

$$\boxed{(s^3 + 2s^2 - s - 2)}\, Y(s) = \left(\frac{3}{s^2+9} + 1\right) \longrightarrow \frac{s^2+12}{s^2+9}$$