

$$\boxed{(\lambda^3 + 2\lambda^2 - \lambda - 2)} Y(\lambda) = \left(\frac{3}{\lambda^2 + 9} + 1 \right) \longrightarrow \frac{\lambda^2 + 12}{\lambda^2 + 9}$$

$$Y(\lambda) = \frac{\lambda^2 + 12}{(\lambda^3 + 2\lambda^2 - \lambda - 2)(\lambda^2 + 9)}$$

$$= \frac{\lambda^2 + 12}{[\lambda^2(\lambda + 2) - (\lambda + 2)](\lambda^2 + 9)}$$

$$Y(\lambda) = \frac{\lambda^2 + 12}{(\lambda - 1)(\lambda + 1)(\lambda + 2)(\lambda^2 + 9)}$$

$$y = \mathcal{L}^{-1} \{ Y(\lambda) \}$$

$$\frac{\lambda^2 + 12}{(\lambda - 1)(\lambda + 1)(\lambda + 2)(\lambda^2 + 9)} = \frac{A}{\lambda - 1} + \frac{B}{\lambda + 1} + \frac{C}{\lambda + 2} + \frac{E\lambda + F}{\lambda^2 + 9}$$

$$\boxed{A = \frac{13}{60}}; \quad \boxed{B = -\frac{13}{20}}; \quad C = \frac{16}{-3 \cdot (-1) \cdot 13} = \boxed{\frac{16}{39}}$$

$$\lambda^2 + 12 = \boxed{A}(\lambda + 1)(\lambda + 2)(\lambda^2 + 9) + \boxed{B}(\lambda - 1)(\lambda + 2)(\lambda^2 + 9) + \boxed{C}(\lambda - 1)(\lambda + 1)(\lambda^2 + 9) + (E\lambda + F)(\lambda - 1)(\lambda + 1)(\lambda + 2)$$

Plug in $\lambda = 0$

$$12 = \frac{13}{60} \cdot 18 - \frac{13}{20} \cdot (-18) + \frac{16}{39} \cdot (-9) + F \cdot (-2)$$

$$\longrightarrow \boxed{F = -\frac{3}{65}}$$

Plug in $\lambda = 2$:

$$16 = \frac{13}{60} \cdot 156 - \frac{13}{20} \cdot 52 + \frac{16}{39} \cdot 39 + \left(2E - \frac{3}{65} \right) \cdot (12) = 0$$

$$2E - \frac{3}{65} = 0$$

$$E = \frac{3}{130}$$

$$y = \frac{13}{60} \mathcal{L}^{-1} \left\{ \frac{1}{\lambda - 1} \right\} - \frac{13}{20} \mathcal{L}^{-1} \left\{ \frac{1}{\lambda + 1} \right\} + \frac{16}{39} \mathcal{L}^{-1} \left\{ \frac{1}{\lambda + 2} \right\}$$

$$+ \frac{3}{130} \mathcal{L}^{-1} \left\{ \frac{\lambda}{\lambda^2 + 9} \right\} - \frac{1}{65} \mathcal{L}^{-1} \left\{ \frac{3}{\lambda^2 + 9} \right\}$$

$$\boxed{y = \frac{13}{60} e^x - \frac{13}{20} e^{-x} + \frac{16}{39} e^{-2x} + \frac{3}{130} \cos(3x) - \frac{1}{65} \sin(3x)}$$

Practice #3

$$y'' + 3xy' - y = 0 \quad ; \quad y(0) = 2 \quad ; \quad y'(0) = 0$$