Practice #

$$y'' + 3xy' - y = 0 \quad ; \quad y(0) = 2 \; ; \; y'(0) = 0$$

Assume: $y = \sum_{n=0}^{\infty} a_n x^n \; ; \; y' = \sum_{n=1}^{\infty} a_n n x^{n-1} \; ; \; y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} + \sum_{n=1}^{\infty} 3a_n n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

k

$$\sum_{K=0}^{\infty} a_{k+2}(k+2)(k+1)x^k$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} 3a_n n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$2a_2 + \sum_{n=1}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} 3a_n n x^n - a_0 - \sum_{n=1}^{\infty} a_n x^n = 0$$

$$(2a_2 - a_0) + \sum_{n=1}^{\infty} \left[a_{n+2}(n+2)(n+1) + 3na_n - a_n\right] x^n = 0$$

$$2a_2 - a_0 = 0 \longrightarrow \boxed{a_2 = \frac{a_0}{2}}$$

$$a_{n+2}(n+2)(n+1) + 3na_n - a_n = 0$$

$$a_{n+2} = \frac{-3na_n + a_n}{(n+2)(n+1)} \rightarrow \boxed{a_{n+2} = \frac{(-3n+1)}{(n+2)(n+1)} a_n \; ; \; n = 1, 2, 3 \ldots}$$

$n=1$: $a_3 = \frac{-2}{3 \cdot 2} a_1 = -\frac{1}{3} a_1$

$n=2$: $a_4 = \frac{-5}{12} a_2 = -\frac{5}{12} \cdot \frac{a_0}{2} = -\frac{5}{24} a_0$

$n=3$: $a_5 = \frac{-8}{20} a_3 = -\frac{8}{20} \cdot \left(-\frac{1}{3}\right) a_1 = \frac{2}{15} a_1$

$n=4$:

$a_6 = \frac{-11}{30} a_4$

$= -\frac{11}{30} \cdot \left(-\frac{5}{24} a_0\right)$

$a_6 = \frac{11}{144} a_0$

Solution: $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \ldots$

$$y = a_0 + a_1 x + \frac{a_0}{2} x^2 - \frac{1}{3} a_1 x^3 - \frac{5}{24} a_0 x^4 + \frac{2}{15} a_1 x^5 + \ldots$$

$$y = a_0\left(1 + \frac{x^2}{2} - \frac{5}{24} x^4 \ldots\right) + a_1\left(x - \frac{1}{3} x^3 + \frac{2}{15} x^5 \ldots\right)$$

$\frac{11}{144} x^6$

$$y(0) = 2 \longrightarrow \boxed{2 = a_0}$$

$$y' = a_0\left(x - \frac{5}{6} x^3 \ldots\right) + a_1\left(1 - x^2 + \frac{2}{3} x^4\right)$$

$$y'(0) = 0 \longrightarrow \boxed{0 = a_1}$$

$$y = 2\left(1 + \frac{x^2}{2} - \frac{5}{24} x^4 + \frac{11}{144} x^6 - \ldots\right)$$

HW11.2

$y = a_0 + a_1 x + a_2 x^2 + \ldots$