HW11.2

$$y = a_0 + a_1 x + a_2 x^2 + \dots$$

a_0

$$y = -a_0 x^2 + \frac{a_0}{2!} x^4 - \frac{a_0}{3!} x^6 + \frac{a_0}{4!} x^8 - \dots$$

$$= -a_0 \left(1 + x^2 - \frac{x^4}{2!} + \frac{x^6}{3!} - \frac{x^8}{4!} + \dots \right)$$

$1 \qquad -\frac{1}{2!} \qquad \frac{1}{3!} \qquad -\frac{1}{4!}$

$$(-1)^{n+1} \frac{1}{n!} x^{2n}$$

$$= -a_0 \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!} x^{2n}$$

Practice Test # 1

$$y'' - 2y' + 2y = \boxed{e^{2x}(\cos x - 3\sin x)}$$

Step 1: $y'' - 2y' + 2y = 0$

$$m^2 - 2m + 2 = 0 \longrightarrow m = 1 \pm i$$

$$y_c = e^{x}\left(c_1 \cos x + c_2 \sin x\right)$$

Step 2: $y_p = e^{2x}\left(A\cos x + B\sin x\right)$

$$y_p' = 2e^{2x}(A\cos x + B\sin x) + e^{2x}(-A\sin x + B\cos x)$$

$$= 2e^{2x} A\cos x + 2e^{2x} B\sin x - e^{2x} A\sin x + e^{2x} B\cos x$$

$$= e^{2x}\left[(2A+B)\cos x + (2B-A)\sin x\right]$$

$$y_p'' = 2e^{2x}\left[(2A+B)\cos x + (2B-A)\sin x\right] + e^{2x}\left[-(2A+B)\sin x + (2B-A)\cos x\right]$$

$$= e^{2x}\left[(4A+2B+2B-A)\cos x + (4B-2A-2A-B)\sin x\right]$$

$$= e^{2x}\left[(3A+4B)\cos x + (-4A+3B)\sin x\right]$$

$$e^{2x}\left[(3A+4B)\cos x + (-4A+3B)\sin x\right] - 2e^{2x}\left[(2A+B)\cos x + (2B-A)\sin x\right] + 2e^{2x}\left[A\cos + B\sin x\right] = e^{2x}\left[\cos x - 3\sin x\right]$$

$$e^{2x}\left[(3A+4B-4A-2B+2A)\cos x + (-4A+3B-4B+2A+2B)\sin x\right] = e^{2x}\left[\cos x - 3\sin x\right]$$

$$(A+2B)\cos x + (-2A+B)\sin x = \cos x - 3\sin x$$

$$A + 2B = 1 \; ; \qquad -2A + B = -3$$

$$A = 7/5 \qquad B = -1/5$$