$$y_p = e^{2x}\left(\frac{7}{5}\cos x - \frac{1}{5}\sin x\right)$$

$$y = y_c + y_p = \ldots$$

HW 10. 4.2 $\quad y'' + 4y = \csc^2(2x)$

Step 1: $\quad y'' + 4y = 0 \rightarrow m^2 + 4 = 0 \rightarrow m = \pm 2i$

$$y_c = c_1 \cos(2x) + c_2 \sin(2x)$$

Step 2: $\quad y_1 = \cos(2x) \; ; \quad y_2 = \sin(2x)$

$$W = \begin{vmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{vmatrix} = 2\cos^2(2x) + 2\sin^2(2x) = \boxed{2}$$

$$y_p = -y_1 \int \frac{y_2\, g(x)}{W}\, dx + y_2 \int \frac{y_1\, g(x)}{W}\, dx$$

$$= -\cos(2x) \int \frac{\sin(2x)\csc^2(2x)}{2}\, dx + \sin(2x) \int \frac{\cos(2x)\cdot\csc^2(2x)}{2}\, dx$$

$$= -\frac{\cos(2x)}{2} \int \cancel{\sin}(2x)\cdot\frac{1}{\sin^{\cancel{2}}(2x)}\, dx + \frac{\sin(2x)}{2} \int \frac{\cos(2x)}{\sin^2(2x)}\, dx$$

$$= -\frac{\cos(2x)}{2} \int \csc(2x)\, dx + \frac{\sin(2x)}{2} \int \frac{\cos(2x)}{\sin^2(2x)}\, dx$$

$$= -\frac{\cos(2x)}{4} \cdot \ln\left|\csc(2x) + \cot(2x)\right| + \frac{\sin(2x)}{2} \int \cot(2x)\csc(2x)\, dx$$

$$= -\frac{\cos(2x)}{4} \ln\left|\csc(2x) + \cot(2x)\right| - \frac{\cancel{\sin(2x)}}{4} \cancel{\csc(2x)}$$

$$y_p = \boxed{-\frac{\cos(2x)}{4} \ln\left|\csc(2x) + \cot(2x)\right| - \frac{1}{4}}$$

$$y = y_c + y_p.$$

(4) $$\int_0^\infty e^{-sx}\, x^2 e^{-2x}\, dx = \int_0^\infty x^2 \cdot e^{-(s+2)x}\, dx$$

$$= \lim_{b\to\infty} \int_0^b x^2 \cdot e^{-(s+2)x}\, dx \; ; \qquad \int x^2 e^{-(s+2)x}\, dx$$