3.2. Polynomial Functions and their graphs

A polynomial function is a function of the form

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

where $a_n ; a_{n-1} ; \dots , a_1 , a_0$ are real numbers

and $a_n \neq 0$.

E.g. $f(x) = -3x^5 + 4x^4 - \frac{1}{2}x^3 + \frac{7}{8}x^2 - x + 10.$

$f(x) = 3x^4 (x-2)(x+5)$

These are examples of polynomial functions.

E.g. $f(x) = -\frac{3}{x^2} + \frac{2}{x} - 5$

$f(x) = 3\sqrt{x} + 5\sqrt[3]{x} + 7$

These are NOT polynomial functions.

Important Terminology:

Given a polynomial function:

$f(x) = \boxed{a_n} x^{\boxed{n}} + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

degree

leading coefficient

The highest exponent of x is called the degree.

The coeff. of x^{degree} is called the leading coefficient.

The term $a_n x^n$ is called the leading term.

E.g. $f(x) = \boxed{3}x^{\boxed{4}} - 7x^3 + 4x^2 - 5x + 10.$

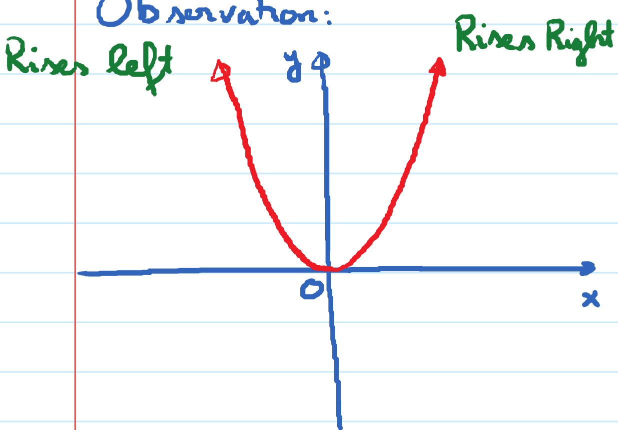
Degree $= 4$

Leading coefficient $= 3$

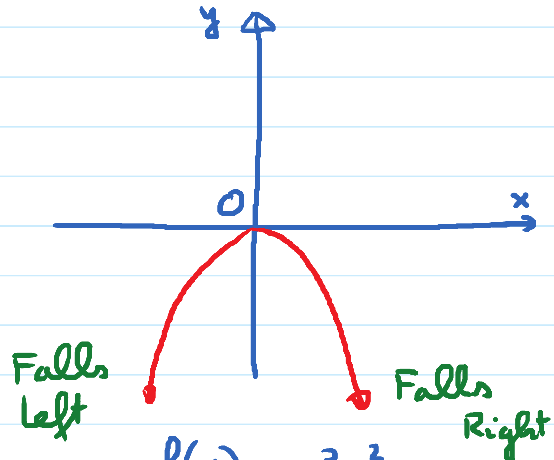
leading term $= 3x^4$

Obj 1: Determine the End Behavior of Polynomial Functions Using the Leading Term Test.

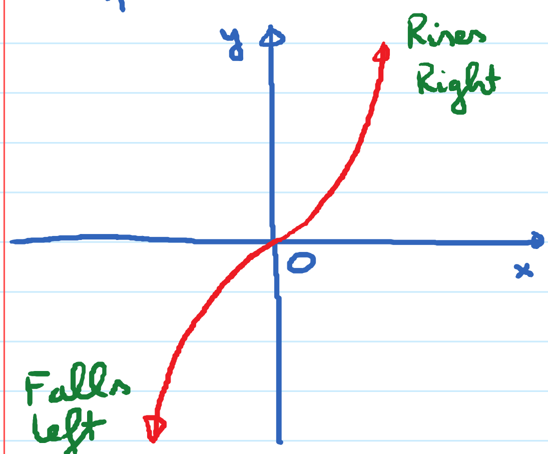
Observation:



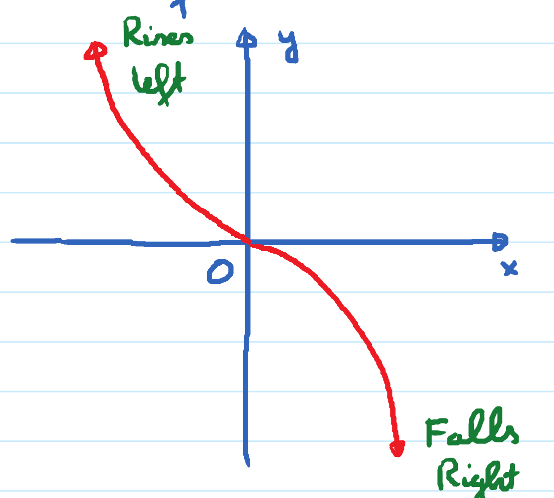
$f(x) = 3x^2$



$f(x) = -3x^2$

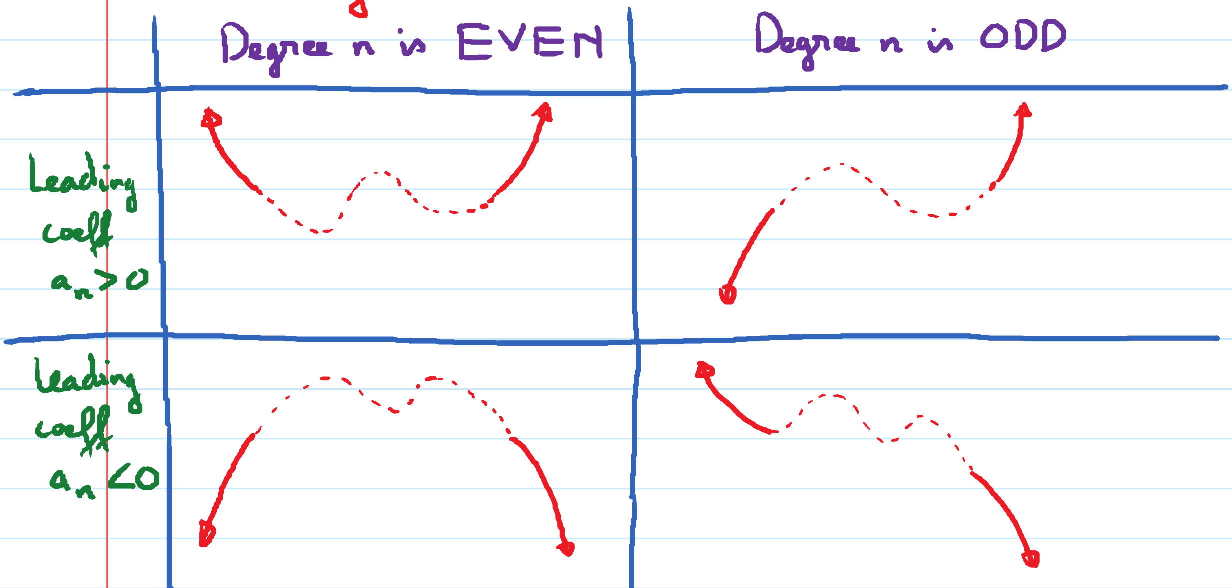


$f(x) = 7x^3$



$f(x) = -4x^3$

The leading term test



A shorter table: Look at leading term $a_n x^n$

	n Even	n Odd
$a_n > 0$	↖ ↗	↙ ↗
$a_n < 0$	↙ ↘	↖ ↘

E.g. Use the leading term test to determine the end behavior of the polynomial function.

(a) $f(x) = 8x^3 + 12x^2 - x - 3$.

(b) $f(x) = -\frac{3}{5}x^4 - 7x^2$.

Sol.

(a) leading term $= \boxed{8}x^{\boxed{3}}$ odd

>0

End Behavior: Falls left, rises right.

(b) leading term $= \boxed{-\frac{3}{5}}x^{\boxed{4}}$ even

<0

End Behavior: Falls left, Falls Right.

E.g. Given $f(x) = -4x^3(x-1)^2(x+5)$

Q1: What is the leading term of f?

Q2: Determine the end behavior of f?

Sol:

Q1: Leading term $= (-4x^3)\cdot(x^2)\cdot(x)$

$= \underset{<0}{\boxed{-4}}x^{\boxed{6}} \rightarrow$ even

Q2:

Falls left, Falls right.

E.g. Given $f(x) = 2x^3(3x-1)(2x+5)^2$

Q1: Find the leading term.

Q2: Determine end behavior.

Sol. Q1: $(2x^3)\cdot(3x)\cdot(2x)^2$

$= 2x^3 \cdot 3x \cdot 4x^2 = 24x^6$

Q2: Rises left, Rises Right.

Obj 2: Zeros of Polynomial Functions and their multiplicity.

Terminology: If f is a polynomial function, the value(s) of x for which $f(x)$ is equal to 0 are called the zeros (or roots) of f.