3.3. Dividing Polynomials, Remainder Theorem and Factor Theorem

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Obj 1: Divide Polynomials using Synthetic Division.

Note: Synthetic Division is used to divide a polynomial $f(x)$ by $x - c$ or $x + c$ (c is a constant)

E.g. Use synthetic division to divide:

$$\underbrace{\boxed{x^3 + 4x^2 - 5x + 5}}_{\text{Dividend}} \text{ by } \underbrace{\boxed{x - 3}}_{\text{Divisor}}$$

$$\begin{array}{c|cccc} & 1 & 4 & -5 & 5 \\ 3 & & 3 & 21 & 48 \\ \hline & 1 & 7 & 16 & \boxed{53} \end{array}$$

multiply

multiply

multiply

$\boxed{53} \longrightarrow$ Remainder.

$\underbrace{1 \quad 7 \quad 16}$ coefficients of Quotient

Quotient $= \boxed{1 \cdot x^2 + 7 \cdot x + 16}$

Result of the division: Quotient $= x^2 + 7x + 16$

Remainder $= 53$.

How to write the result?

1st way:

$$\underbrace{\boxed{x^3 + 4x^2 - 5x + 5}}_{\text{Dividend}} = \underbrace{\boxed{(x^2 + 7x + 16)}}_{\text{Quotient}} \cdot \underbrace{\boxed{(x - 3)}}_{\text{Divisor}} + \underbrace{\boxed{53}}_{\text{Remainder}}$$

2nd way:

$$\frac{\overset{\text{Dividend}}{\boxed{x^3 + 4x^2 - 5x + 5}}}{\underset{\text{Divisor}}{\boxed{x - 3}}} = \overset{\text{Quotient}}{\boxed{x^2 + 7x + 16}} + \frac{\overset{\text{Remainder}}{\boxed{53}}}{\boxed{x - 3}} \text{ divisor}$$

E.g. Use synthetic division to divide

$x^3 - 7x - 6$ by $x + 2$ (Note: x^2 term is missing, we put 0 as the coefficient.)

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -7 & -6 \\ & & -2 & 4 & 6 \\ \hline & 1 & -2 & -3 & \boxed{0} \end{array} \longrightarrow \text{Remainder}$$

$\boxed{1x^2 - 2x - 3} \longrightarrow$ Quotient.

$$\boxed{x^3 - 7x - 6} = \boxed{(x^2 - 2x - 3)} \cdot \boxed{(x+2)} + \boxed{0}$$

Dividend = Quotient · Divisor + Remainder

$$x^3 - 7x - 6 = (x^2 - 2x - 3)(x+2)$$

Obj 2: The Remainder Theorem.

The Remainder Theorem:

If the polynomial $f(x)$ is divided by $x - c$, then the remainder is equal to $f(c)$.

If the polynomial $f(x)$ is divided by $x + c$, then the remainder is equal to $f(-c)$.

E.g. when we divide $x^3 + 4x^2 - 5x + 5$ by $x - 3$, we obtained the remainder = 53.

The remainder theorem asserts that if we plug 3 into $\underbrace{x^3+4x^2-5x+5}_{f(x)}$; i.e., if we calculate $f(3)$, we will get 53.

Let's check: $f(3) = (3)^3 + 4(3)^2 - 5(3) + 5$

$= 27 + 4\cdot 9 - 15 + 5$

$= 27 + 36 - 15 + 5 = \boxed{53}$.

Obj 3: The Factor Theorem

Let $f(x)$ be any polynomial.

(a) If $f(c) = 0$, then $x - c$ is a factor of $f(x)$

(b) If $x - c$ is a factor of $f(x)$, then $f(c) = 0$

E.g. Consider the equation:

$$\underbrace{\boxed{2x^3 - 3x^2 - 11x + 6}}_{f(x)} = 0$$

Given that $x = 3$ is a solution. Find the remaining solutions.

Sol: Since $x = 3$ is a solution, $f(3) = 0$.

The Factor Theorem says that $x - 3$ must be a factor of $f(x)$.

This means: $f(x) = (x-3) \cdot (\text{something})$

If we divide $f(x)$ by $x - 3$, we will find the other factor.

Set that other factor equal to 0 will help us find the remaining solutions.

$$(2x^3 - 3x^2 - 11x + 6) \div (x - 3)$$

$$\begin{array}{r|rrrr} & 2 & -3 & -11 & 6 \\ 3 & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$\underbrace{2 \quad 3 \quad -2}_{\text{Quotient } = 2x^2 + 3x - 2}$

This quotient is the remaining factor:

$$2x^3 - 3x^2 - 11x + 6 = (2x^2 + 3x - 2) \cdot (x - 3)$$

To find the remaining zeros, we set quotient $= 0$

$$2x^2 + 3x - 2 = 0$$

$$(2x - 1)(x + 2) = 0 \quad (\text{Factor})$$

$2x - 1 = 0 \quad ; \quad x + 2 = 0$

$x = \frac{1}{2} \quad ; \quad x = -2$

Solution set: $\boxed{\left\{ 3;\ \frac{1}{2}\ ;\ -2 \right\}}$