4.1. Exponential Functions

Tuesday, November 19, 2019 9:45 AM

Obj 1: Exponential Functions and their graphs

The exponential function with base b is the function of the form:

$$f(x) = b^x \quad \text{or} \quad y = b^x$$

Here: b is a constant, b is positive ($b > 0$)

$b \neq 1$.

b is called the base of the function.

E.g. $f(x) = 2^x$; $f(x) = \left(\frac{1}{2}\right)^x$

base = 2 ; base = $\frac{1}{2}$

$f(x) = (2.71828)^x$

base

$f(x) = 3^{x+1}$; $f(x) = 12 \cdot 3^{x-1}$

base = 3 ; base = 3

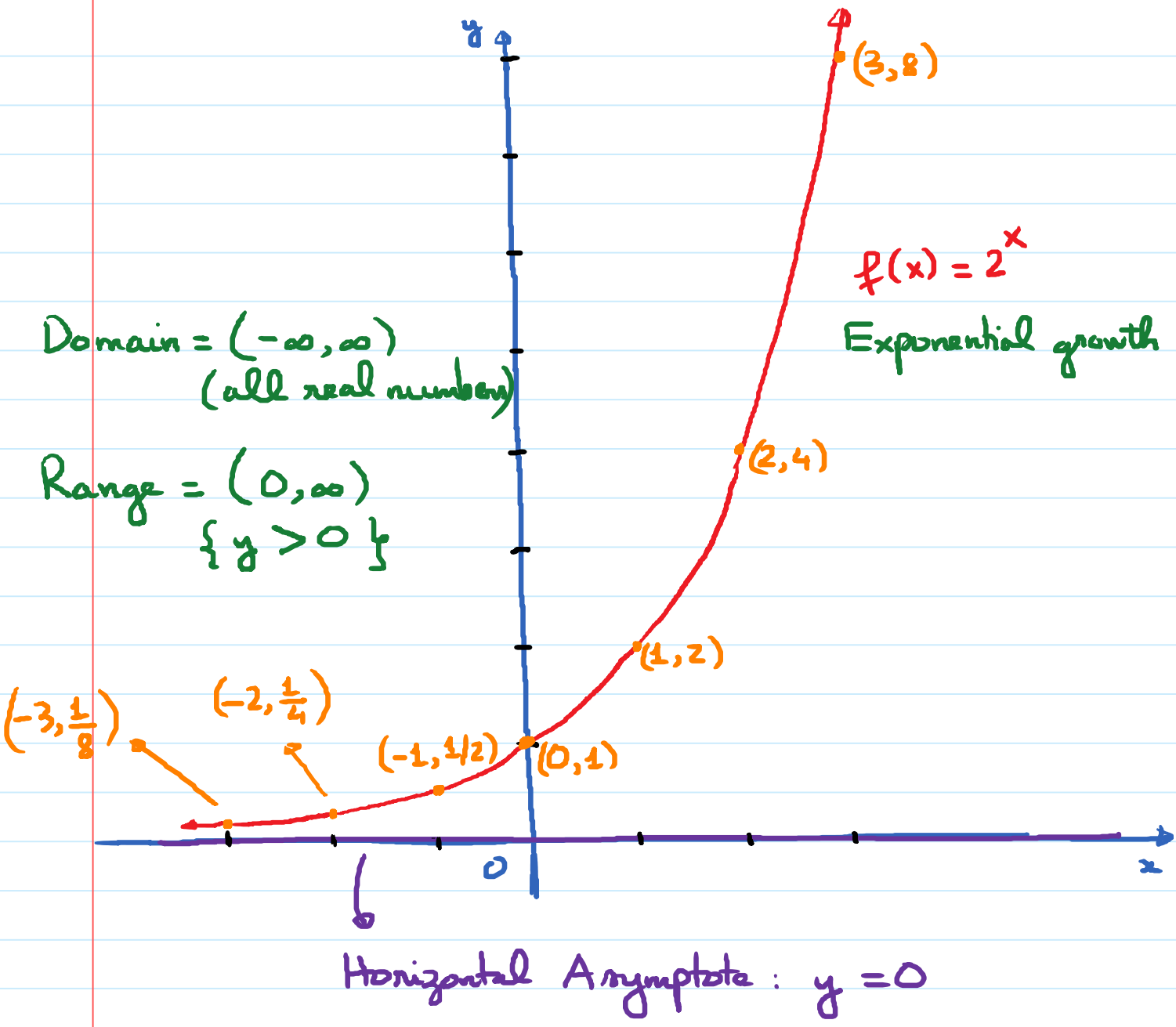
These are examples of exponential function.

Note: In calculator, use $\wedge$ to do exponents.

* Graphs of Exponential functions

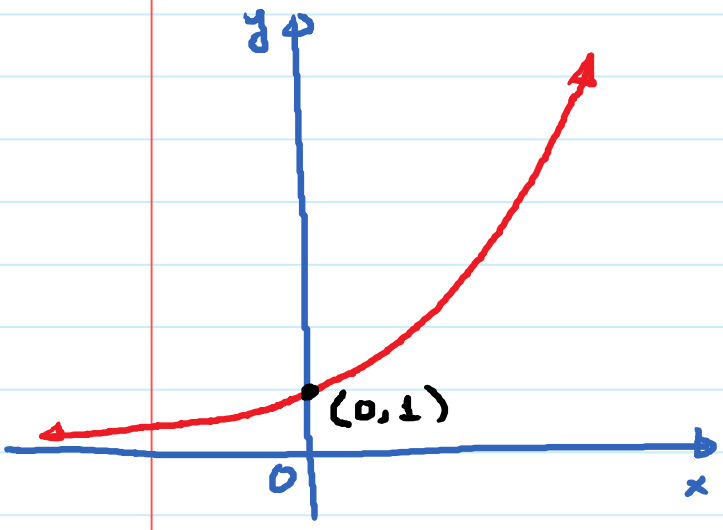
E.g. Consider the function $f(x) = 2^x$

x	$f(x) = 2^x$	Point (x, y)
-3	$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$(-3, \frac{1}{8})$
-2	$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$(-2, \frac{1}{4})$
-1	$f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$	$(-1, \frac{1}{2})$
0	$f(0) = 2^0 = 1$	$(0, 1)$
1	$f(1) = 2^1 = 2$	$(1, 2)$
2	$f(2) = 2^2 = 4$	$(2, 4)$
3	$f(2) = 2^3 = 8$	$(3, 8)$


y
x
0
(3,8)
(2,4)
(1,2)
(0,1)
(-1,1/2)
(-2,1/4)
(-3,1/8)
f(x) = 2^x
Exponential growth
Domain = (-∞, ∞)
(all real numbers)
Range = (0, ∞)
{y > 0}
Horizontal Asymptote : y = 0

In general, the graph of $f(x) = b^x$ has the following properties:

Base $b > 1$



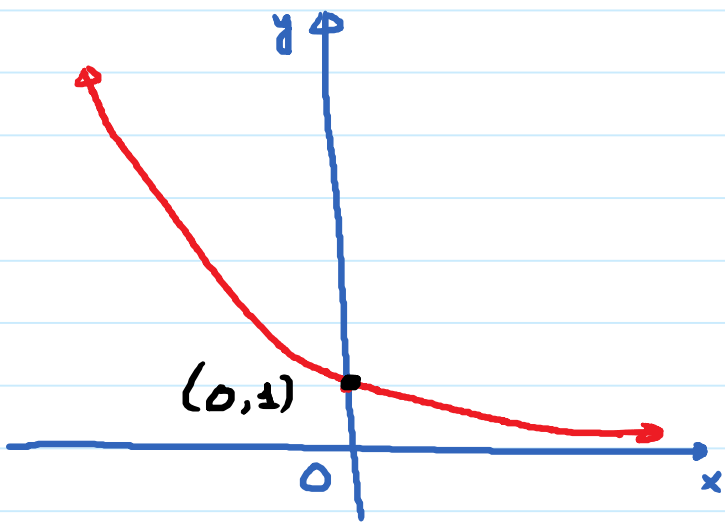
Horizontal Asymptote : $y = 0$

Domain : $(-\infty, \infty)$

Range : $(0, \infty)$

Exponential growth

Base $0 < b < 1$



$y = 0$

$(-\infty, \infty)$

$(0, \infty)$

Exponential Decay

Obj 2: Transformations of Exponential Functions.

Transformation	Equation	Description
Vertical Translation	$y = b^x + c$	up, c units
	$y = b^x - c$	down, c units
Horizontal Translation	$y = b^{x+c}$	left, c units
	$y = b^{x-c}$	Right, c units
Vertical Stretch/Shrink	$y = c \cdot b^x$	$c > 1$: stretch
		$0 < c < 1$: shrink
Reflection	$y = -b^x$	Flip across x-axis

E.g. Describe the transformation.

Find domain, range, asymptote(s), y-intercept of the given function.

(a) $f(x) = 2^x - 7$.

Parent function: $y = 2^x$.

Down, 7 units.

Domain: $(-\infty, \infty)$

Range: $(-7, \infty)$

Horizontal asymptote: $y = -7$

y-intercept: $(0, -6)$ (Plug $x = 0$ into f)

(b) $g(x) = -3^{x+1}$

Parent function: $y = 3^x$

Shift left 1, flip across x.

Domain: $(-\infty, \infty)$; Range: $(-\infty, 0)$

H. A. : $y = 0$

y-intercept : $y = -3^{0+1} = -3$.

$(0, -3)$

Obj 3: The natural base e.

$$e \simeq 2.718281827\ldots.$$

In many applications of exponential function, the base of the function equal to this special number called e.

Graph of $f(x) = e^x$ is an exp. growth because $e > 1$.