4.2. Logarithmic (Log) Functions.

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Obj 1: Definition of Logarithmic Functions.

The logarithmic function with base b is the function written as:

$$f(x) = \log_b x$$ (read as log base b of x)

$\log_b x$ gives you the exponent y so that $b^y = x$

In other words,

$\boxed{y} = \log_{\boxed{b}} \boxed{x}$ is equivalent to $\boxed{b}^{\boxed{y}} = \boxed{x}$

exponent, base, number — exponent, base, number

E.g. Function: $f(x) = \log_2 x$

$f(4) = \log_{\boxed{2}} \boxed{4} = \boxed{2}$ $(2^{2} = 4)$

number, base, exp.

(read as log base 2 of 4)

$f(8) = \log_{\boxed{2}} \boxed{8} = \boxed{3}$ $(2^3 = 8)$

number, base, exp

$f(16) = \log_2 16 = 4$

$f(32) = \log_2 32 = 5$

$f(64) = \log_2 64 = 6$

Eg. $f(x) = \log_3 x$.

$f(27) = \log_3 27 = 3$ (b/c $3^3 = 27$)

$f(81) = \log_3 81 = 4$ (b/c $3^4 = 81$)

Obj 2: Change from logarithmic form to exponential form and vice versa.

exponent number

Logarithmic form: $\boxed{y} = \log_{\boxed{b}} \boxed{x}$

base

exponent

Exponential form: $\boxed{b}^{\boxed{y}} = \boxed{x}$

number

base

Eg. Write the equation in exponential form.

(a) $2 = \log_5 x \longrightarrow$ Exp. form: $\boxed{5^2 = x}$

(b) $\log_3 7 = y \longrightarrow \boxed{3^y = 7}$

E.g. Write each equation in logarithmic form.

(a) $b^3 = 27 \longrightarrow$ Log form: $\boxed{\log_b 27 = 3}$

(b) $2^5 = x \longrightarrow$ Log form: $\boxed{\log_2 x = 5}$

Obj 3: Basic Properties of Log function.

(1) $\log_b b = 1$ (because $b^{\overset{\text{exp.}}{\boxed{1}}} = b$)

(2) $\log_b 1 = 0$ (because $b^{\overset{\text{exp}}{\boxed{0}}} = 1$)

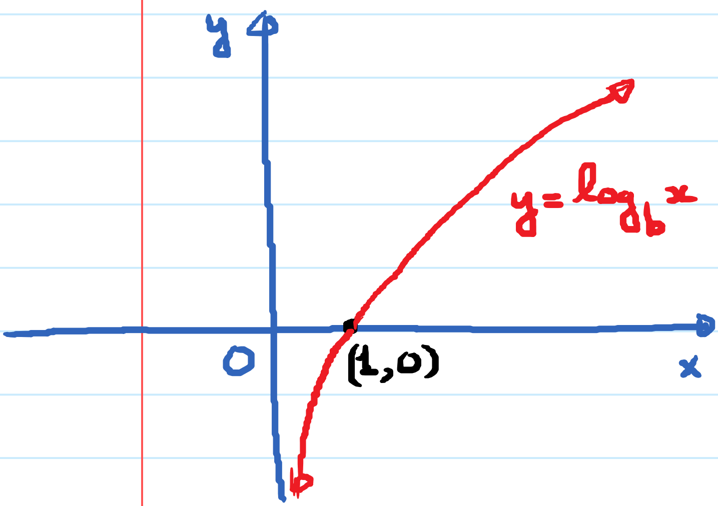
Inverse Properties of Logarithms.

(3) $\log_b b^x = x$ E.g. $\log_7 7^8 = 8$

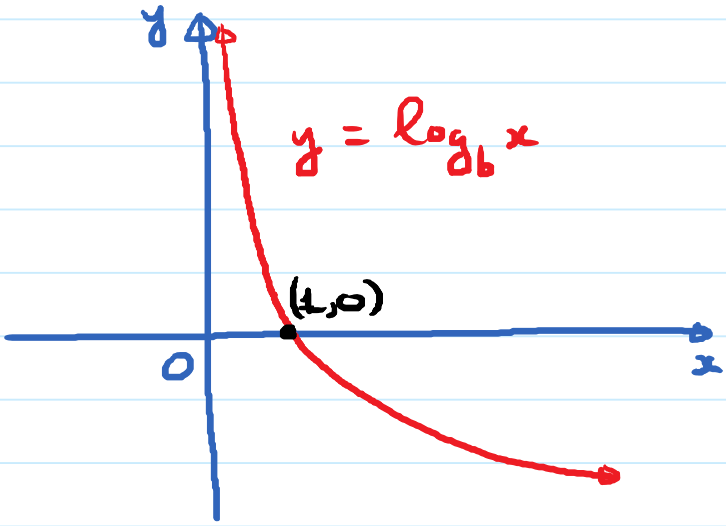
(4) $b^{\log_b x} = x$ E.g. $3^{\log_3 17} = 17$

Obj 4: Graphs of logarithmic functions.

Base $b > 1$



Base $0 < b < 1$



	Base $b > 1$	Base $0 < b < 1$
Vertical Asymptote:	$\boxed{x = 0}$	$\boxed{x = 0}$
Domain:	$(0, \infty)$	$(0, \infty)$
Range:	$(-\infty, \infty)$	$(-\infty, \infty)$

* Graphs Transformations.

Parent function : $y = \log_b x$.

$y = \log_b x + c$: up, c units

$y = \log_b x - c$: down, c units

$y = \log_b(x+c)$: left, c units

$y = \log_b(x-c)$: right, c units

$y = c \cdot \log_b x$ $\begin{cases} c > 1: \text{ vertical stretch} \\ 0 < c < 1: \text{ vertical shrink} \end{cases}$

$y = -\log_b x$: flip over x-axis.

Obj 5: The domain of a logarithmic function.

E.g. Find domain of $y = \log_2(x+3)$

Domain $= (-3, \infty)$

E.g. Find domain of $y = \log_4(x-5)$

Domain $= (5, \infty)$

More generally, to find domain of

$y = \log_b(\text{expression})$

We solve the inequality: $\boxed{\text{expression} > 0}$

E.g. Find domain of $f(x) = \log_3(\boxed{7x - 21})$

Set: $7x - 21 > 0$

$7x > 21$

$x > 3.$

$\boxed{\text{Domain} = (3, \infty)}$

Obj 6: Natural Logarithms and Common Logarithms.

* The logarithmic function with base e is called the natural logarithmic function.

We write $\ln(x)$ instead of $\log_e x$

E.g. $\ln(4) \simeq 1.386$ $\quad \ln(e) = 1$

$\ln(1) = 0$ $\quad \ln(e^2) = 2$

Note: $\boxed{\ln(e^x) = x}$

The logarithmic function with base 10 is called the common logarithmic function.

We write $\log x$ instead of $\log_{10} x$

E.g.

$\log(7) \simeq 0.845$; $\log(10) = 1$

$\log(1) = 0$; $\log(100) = 2$

$\log(1000) = 3$

Note: $\boxed{\log(10^x) = x}$

Obj 7: Change of base formula - using natural log or common log to evaluate log of any base.

E.g. $\log_2 8 = 3$; $\dfrac{\ln(8)}{\ln(2)} = 3$, $\dfrac{\log(8)}{\log(2)} = 3$.

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$\log_3(9) = 2$; $\dfrac{\ln(9)}{\ln(3)} = 2$; $\dfrac{\log(9)}{\log(3)} = 2$

In general,

$\log_a b =$

- use natural log : $\dfrac{\ln(b)}{\ln(a)}$
- use common log : $\dfrac{\log(b)}{\log(a)}$
- use any base : $\dfrac{\log_c(b)}{\log_c(a)}$

E.g. $\log_7 18$

- $\dfrac{\ln(18)}{\ln(7)}$
- $\dfrac{\log(18)}{\log(7)}$

$\simeq 1.485$