4.3. Properties of Logarithms

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Properties of Logarithms.

(1) Product Rule

$$\boxed{\log_b(M \cdot N) = \log_b(M) + \log_b(N)}$$

E.g. $\log_2(4 \cdot 7) = \log_2(4) + \log_2(7)$

$\ln(7 \cdot x) = \ln(7) + \ln(x)$

(2) Quotient Rule

$$\boxed{\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)}$$

E.g. $\log\left(\frac{x}{2}\right) = \log(x) - \log(2)$

$\ln\left(\frac{7}{y}\right) = \ln(7) - \ln(y)$

(3) Power Rule

$$\boxed{\log_b(M^p) = p \cdot \log_b(M)}$$

E.g. $\log_3(x^{\boxed{2}}) = 2 \cdot \log_3(x)$

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E.g. $\ln(\sqrt{x}) \overset{\text{Rewrite}}{=} \ln(x^{\boxed{\frac{1}{2}}}) \overset{\text{Power Rule}}{=} \frac{1}{2}\cdot\ln(x)$

$\log(\sqrt[3]{7x}) \underset{\text{Rewrite}}{=} \log(7x)^{\boxed{\frac{1}{3}}} \underset{\text{Power Rule}}{=} \frac{1}{3}\log(7x)$

Obj 1: Expand Logarithmic Expressions.

E.g. Expand the expression

(a) $\log_7(x^2\cdot y^3) \overset{\text{product rule}}{=} \log_7(x^2) + \log_7(y^3)$

$\overset{\text{power rule}}{=} \boxed{2\cdot\log_7 x + 3\cdot\log_7 y}$

(b) $\ln(x^4\sqrt[3]{y}) \overset{\text{product rule}}{=} \ln(x^4) + \ln(y^{\frac{1}{3}})$

$\overset{\text{power rule}}{=} \boxed{4\cdot\ln x + \frac{1}{3}\ln y}$

(c) $\log_2\left(\frac{x^3 y}{z^2}\right) \overset{\text{quotient rule}}{=} \log_2(x^3y) - \log_2(z^2)$

$\underset{\text{product rule}}{=} \log_2(x^3) + \log_2 y - \log_2(z^2)$

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Power Rule

$$= \boxed{3\log_2(x) + \log_2(y) - 2\log_2(z)}$$

Obj 2: Condense Logarithmic Expressions.

Properties:

$$\boxed{\begin{aligned} \log_b(M) + \log_b(N) &= \log_b(M \cdot N) \\ \log_b(M) - \log_b(N) &= \log_b\left(\frac{M}{N}\right) \\ p \cdot \log_b(M) &= \log_b(M^p) \end{aligned}}$$

E.g. Condense the expression.

quotient rule

(a) $\log(7x+6) - \log x = \log\left(\frac{7x+6}{x}\right)$

Power Rule

(b) $\frac{1}{2}\log x + 4\log(x-1) = \log\left(x^{\frac{1}{2}}\right) + \log(x-1)^4$

Product rule

$$= \boxed{\log\left(\sqrt{x} \cdot (x-1)^4\right) = \log\left((x-1)^4\sqrt{x}\right)}$$

(c) $\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y$

$$= \frac{1}{4}\log_b x - \left(2\log_b 5 + 10\log_b y\right)$$

Power Rule

$$= \log_b x^{\frac{1}{4}} - \left(\underbrace{\log_b 5^2 + \log_b y^{10}}_{\text{Product Rule}}\right)$$

$$= \log_b x^{\frac{1}{4}} - \left(\log_b \left(25y^{10}\right)\right)$$

Quotient Rule

$$= \boxed{\log_b\left(\frac{\sqrt[4]{x}}{25y^{10}}\right)}$$

Obj 3: Change of base formula.

E.g. Find $\log_2 7$ using calculator:

$\log_2 7$

Using ln $= \dfrac{\ln 7}{\ln 2} \approx \boxed{2.807}\ldots$

Using log $= \dfrac{\log 7}{\log 2} \approx \boxed{2.807}\ldots$

Same answer

Using ln $\rightarrow \dfrac{\ln(2506)}{\ln(7)} \approx 4.02$

$\log_7 2506$

Using log $\rightarrow \dfrac{\log(2506)}{\log(7)} \approx 4.02$.

In general,

$$\boxed{\log_b M = \frac{\ln(M)}{\ln(b)} \text{ or } \frac{\log(M)}{\log(b)}}$$

Moreover,

$$\boxed{\log_b M = \frac{\log_c M}{\log_c b} \quad (\text{where } c \text{ is any other base})}$$