4.4. Exponential and Logarithmic Equations

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Exponential Equations

Obj 1: Use like bases to solve exponential equations.

E.g. Solve $2^{3x-8} = 16$

Step 1: Rewrite the equation so that both sides have the same base: $\boxed{b^M = b^N}$

Exp. Exp.

$$2^{\boxed{3x-8}} = 2^{\boxed{4}}$$

Step 2: Set exponents equal: Set $\boxed{M = N}$.

$$3x - 8 = 4$$

Step 3: Solve for x

$$3x = 12 \quad (\text{Add } 8)$$

$$\boxed{x = 4} \quad (\text{Divide by } 3)$$

E.g. Solve

$$27^{x+3} = 9^{x-1}$$

Step 1: Rewrite so that both sides have the same base.

Here, the common base is 3

$$\left(3^3\right)^{x+3} = \left(3^2\right)^{x-1} \qquad \left(27 = 3^3 ;\ 9 = 3^2\right)$$

Recall: Power of power rule:

$$\boxed{\left(b^p\right)^q = b^{p \cdot q}}$$

Apply this here:

$$3^{\overset{\text{Exp}}{\boxed{3\cdot(x+3)}}} = 3^{\overset{\text{Exp}}{\boxed{2\cdot(x-1)}}}$$

Step 2: Set exponents equal

$$3(x+3) = 2(x-1)$$

Step 3: Solve for x

$$3x + 9 = 2x - 2 \qquad \text{(Distribute)}$$

$$\boxed{x = -11} \qquad (\text{Isolate } x)$$

E.g. Solve

$$8^{x+2} = 4^{x-3}$$

$$\left(2^3\right)^{x+2} = \left(2^2\right)^{x-3} \qquad \text{(Rewrite)}$$

$$2^{\boxed{3\cdot(x+2)}} = 2^{\boxed{2\cdot(x-3)}} \qquad \text{(Power of Power)}$$

$$3(x+2) = 2(x-3)$$ (Set exponents equal)

$$3x+6 = 2x-6$$ (Distribute)

$$\boxed{x = -12}$$ (solve for x)

Obj 2: Solve exponential equations using Logarithms

Note: In many cases, it is difficult to rewrite both sides into the same base. We use logarithm in those situations.

E.g. Solve $\underbrace{\boxed{4^x}}_{\text{exp. expression}} - 1 = 14$

Step 1: Isolate the exponential expression.

$$4^x = 15$$ (Add 1 to both sides)

Step 2: Take the $\underbrace{\text{natural logarithm}}_{\ln}$ of both sides

$$\ln(4^x) = \ln(15)$$

Step 3: Apply the Power Rule to simplify

$$x \cdot \underbrace{\boxed{\ln(4)}}_{\text{a number}} = \underbrace{\boxed{\ln(15)}}_{\text{a number}}$$

Step 4: Solve for x.

Exact answer

$$\boxed{x = \frac{\ln(15)}{\ln(4)}}$$ (Divide both sides by $\ln(4)$)

We can use calculator to get decimal approximation for x : $x \approx 1.953...$

E.g. Solve:

exp. expression

$$3 \cdot \boxed{7^{2x+1}} + 2 = 11$$

Step 1: Isolate the exponential expression

$$3 \cdot 7^{2x+1} = 9 \quad \text{(Subtract 2)}$$

$$7^{2x+1} = 3 \quad \text{(Divide by 3)}$$

Step 2: Take ln of both sides

$$\ln(7^{2x+1}) = \ln(3)$$

Step 3: Power Rule :

$$(2x+1)\ln(7) = \ln(3)$$

Step 4: Solve for x:

$$2x + 1 = \frac{\ln(3)}{\ln(7)} \quad \text{(Divide by } \ln(7))$$

$$x = \frac{\ln(3)}{\ln(7)} - 1 \quad \text{(Subtract 1)}$$

$$\boxed{x = \frac{\frac{\ln(3)}{\ln(7)} - 1}{2}} \quad \text{(Divide by 2)}$$

Note: If the base is e, then we have:

$$\boxed{\ln(e^M) = M}$$

E.g. Solve: $5 \cdot e^{x+6} - 12 = 8$

Step 1: Isolate the exponential expression:

$$5 \cdot e^{x+6} = 20 \quad \text{(Add 12)}$$

$$e^{x+6} = 4 \quad \text{(Divide by 5)}$$

Step 2: Take $\ln$ of both sides:

$$\ln(e^{x+6}) = \ln(4)$$