1.5 Quadratic Equations

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E.g. of a quadratic equation:

$$x^2 - 7x + 10 = 0$$

In general, a quadratic equation is an equation of the form

$$ax^2 + bx + c = 0$$

a, b, c are real numbers and $a \neq 0$

E.g. $x^2 - 7x + 10 = 0$

$a = 1$; $b = -7$; $c = 10$

Obj 1: Solve a quadratic equation by factoring.

$x^2 - 7x + 10 = 0 \to$ Hard to solve

$(x-5)(x-2) = 0 \to$ easier to solve

Set $x - 5 = 0$ or $x - 2 = 0$

$x = 5$ or $x = 2$

Claim: $(x-5)(x-2) = x^2 - 7x + 10$

Why? $(x-5)(x-2) = x^2 \underbrace{- 2x - 5x}_{\text{like terms}} + 10$

$= x^2 - 7x + 10$

Solve a quadratic equation by factoring is about going from the first form to the second form.

E.g. (a) $4x^2 - 2x = 0$

$2x(2x - 1) = 0$ (Factor out the common factor $2x$)

$2x = 0$ or $2x - 1 = 0$ (Set each factor equal to 0)

$2x = 0 \rightarrow \boxed{x = 0}$

$2x - 1 = 0 \rightarrow 2x = 1 \rightarrow \boxed{x = \frac{1}{2}}$

Solution set: $\boxed{\left\{0, \frac{1}{2}\right\}}$

(b) $2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$ (Right hand side $= 0$)

$(2x - 1)(x + 4) = 0$ (Factor)

$2x - 1 = 0$ or $x + 4 = 0$ (Set each factor equal to zero)

$\boxed{x = \frac{1}{2}}$ or $\boxed{x = -4}$

Solution set: $\left\{\frac{1}{2}, -4\right\}$

E.g. $2x^2 + x = 1$

$2x^2 + x - 1 = 0$ (Right hand side $= 0$)

$(2x - 1)(x + 1) = 0$ (Factor)

$2x - 1 = 0$ or $x + 1 = 0$

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$x = \frac{1}{2}$ or $x = -1$

Solution set : $\left\{\frac{1}{2}, -1\right\}$

Obj 2: Solve quadratic equations by the square root property.

E.g. $x^2 = 4 \longrightarrow x = \pm 2$

Square Root Property.

Stuff = any algebraic expression.

d = a number.

If $(\text{Stuff})^2 = d$, then

$\text{Stuff} = \sqrt{d}$ or $\text{Stuff} = -\sqrt{d}$

We can write this in an equivalent way as:

$\text{Stuff} = \pm\sqrt{d}$

Note: Before you can apply the square root property, a squared expression must be isolated on one side of the equation.

E.g. (a) $3x^2 - 15 = 0$

$\rightarrow 3x^2 = 15 \rightarrow x^2 = 5$ (Isolate a squared expression on one side)

$\rightarrow \boxed{x = \pm\sqrt{5}} \longrightarrow$ Square root property

Solution set: $\{\sqrt{5}, -\sqrt{5}\}$

(b) $5x^2 + 45 = 0$

$\rightarrow 5x^2 = -45 \rightarrow x^2 = -9$ (Isolate x^2)

$\rightarrow x = \pm\sqrt{-9}$ (Square Root Property)

$\rightarrow x = \pm\sqrt{-1 \cdot 9} = \pm\sqrt{i^2 \cdot 9}$ (Recall that $i^2 = -1$ ↳ imaginary unit)

$\rightarrow x = \pm\sqrt{i^2} \cdot \sqrt{9}$

$\rightarrow x = \pm i \cdot 3 \rightarrow x = \pm 3i$

Solution set: $\{\underbrace{3i, -3i}_{\text{imaginary numbers}}\}$

Stuff

(c) $(\boxed{x-2})^{\boxed{2}} = \boxed{6}$ → number

By the Square Root Property:

$x - 2 = \pm\sqrt{6}$

$x = \pm\sqrt{6} + 2$

(or $x = 2 \pm \sqrt{6}$)

Solution set: $\boxed{\{2+\sqrt{6}, 2-\sqrt{6}\}}$

Obj 3: Solve quadratic equations by using the quadratic formula.

The quadratic formula:

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$; $a \neq 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

E.g. $8x^2 + 2x - 1 = 0$

$a = 8$ $b = 2$ $c = -1$

Quadratic formula:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot(8)\cdot(-1)}}{2\cdot(8)}$$

$$= \frac{-2 \pm \sqrt{36}}{16} = \frac{-2 \pm 6}{16}$$

$x = \frac{-2+6}{16}$ or $x = \frac{-2-6}{16}$

$x = \frac{4}{16} = \frac{1}{4}$ or $x = \frac{-8}{16} = -\frac{1}{2}$

Solution set: $\left\{ \frac{1}{4}, -\frac{1}{2} \right\}$

E.g. Solve using the quadratic formula:

$$2x^2 + 2x - 1 = 0$$

$a = 2$; $b = 2$; $c = -1$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot(2)\cdot(-1)}}{2\cdot(2)}$$

$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm \sqrt{4\cdot 3}}{4} = \frac{-2 \pm \sqrt{4}\cdot\sqrt{3}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4} = \frac{\cancel{2}(-1 \pm \sqrt{3})}{\cancel{4}\ 2}$$

$$x = \frac{-1 \pm \sqrt{3}}{2}$$

Solution set: $\left\{ \frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2} \right\}$

E.g. (Non-real Solutions)

Solve $x^2 - 2x + 2 = 0$ using quadratic formula.

$a = 1$; $b = -2$; $c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot(1)\cdot(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm \sqrt{i^2\cdot 4}}{2}$$

$$= \frac{2 \pm i\cdot 2}{2} = \frac{\cancel{2}(1 \pm i)}{\cancel{2}} = 1 \pm i$$

Solution set: $\boxed{\{1+i, 1-i\}}$