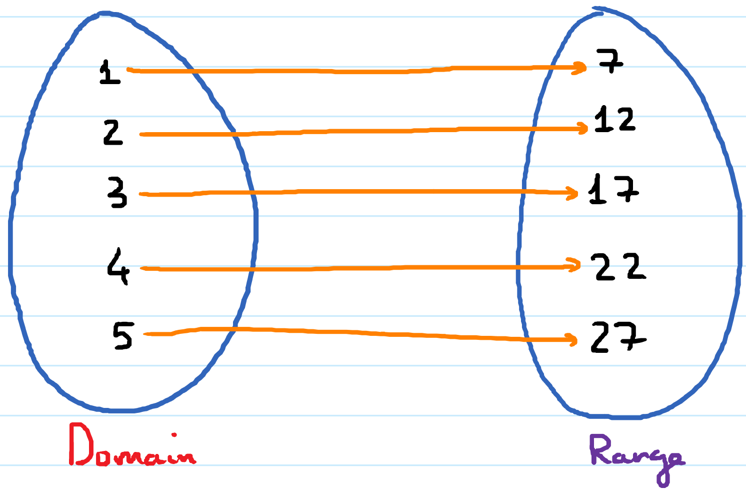
Obj 3: Functions as Equations



This is a function

x represents the # of tickets

y represents the cost of buying x tickets

The equation $y = 5 \cdot x + 2$ gives us the way to calculate the cost of buying x tickets.

This equation: $\boxed{y} = 5\boxed{x} + 2$ defines y as a function of x.

independent variable

dependent variable

Note: Not all equations in y and x define y as a function of x

E.g.

$$x^2 + y^2 = 4.$$

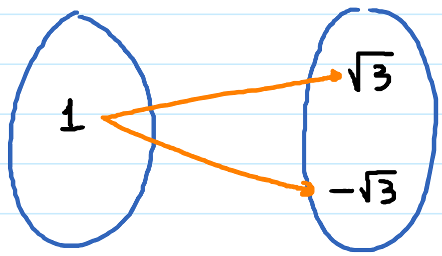
$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4 - x^2}$$

This equation does not define y as a function of x.

Reason: For most values of x, there are 2 values of y that correspond to it.

For example, if $x = 1$; $y = \pm\sqrt{4-(1)^2} = \pm\sqrt{3}$.



Note: If an equation is solved for y and more than one values of y can be obtained from a value of x, then that equation does not define y as a function of x.

E.g. Solve each equation for y and determine whether the equation defines y as a function of x.

(a) $2x + 3y = 6$ (b) $2x^2 + 3y^2 = 1$.

Sol:

(a) $y = \dfrac{6-2x}{3}$. This defines y as a function of x

(b) $3y^2 = 1 - 2x^2 \longrightarrow y^2 = \dfrac{1-2x^2}{3}$

$\longrightarrow y = \pm\sqrt{\dfrac{1-2x^2}{3}}$. This does not define y as a function of x.

Obj 4: Function Notation and Evaluate Functions

When an equation such as $y = 5x + 2$ defines y as a function of x, we can use function notation to describe it.

We name the function by the letter f (or g or h or k or ℓ, etc.)

We rewrite the variable y with the notation $f(x)$.
(This is read as f of x)

So, the equation $y = 5x + 2$ is rewritten in function notation as

$$\underbrace{\overset{\text{name of function}}{\boxed{f}}(x)}_{f \text{ of } x} = 5x + 2$$

x: input
$f(x)$: output

The equation tells us how to get the output from the input.

Evaluate the function at 5 → Replace x by 5

$$\underbrace{f(5)}_{f \text{ of } 5} = 5(5) + 2 = 27$$

input   output

Evaluate the function at 100 → Replace x by 100

$$f(100) = 5(100) + 2 = 502$$

E.g. Evaluate a function.

$$f(x) = x^2 - 2x + 7$$

Evaluate: (a) $f(2)$ (b) $f(0)$ (c) $f(-1)$

Sol:

(a) $f(2) = (2)^2 - 2(2) + 7$

$= 4 - 4 + 7 = 7$

(b) $f(0) = (0)^2 - 2(0) + 7 = 7$

(c) $f(-1) = (-1)^2 - 2(-1) + 7$

$= 1 + 2 + 7 = 10$

Note: The input does not always have to be a number.

E.g. $f(x) = x^2 - 2x + 7$

placeholder

$$f(\boxed{}) = \boxed{}^2 - 2\boxed{} + 7$$

Evaluate $f(a)$?

$$f(a) = (a)^2 - 2(a) + 7 = a^2 - 2a + 7$$

Evaluate $f(a+1)$?

$$f(a+1) = (a+1)^2 - 2(a+1) + 7$$

$$f(a+1) = (a+1)(a+1) - 2(a+1) + 7$$

$$= a^2 + \cancel{a} + \cancel{a} + 1 - \cancel{2}a - 2 + 7$$

$$\boxed{f(a+1) = a^2 + 6}$$

Evaluate $f(x+2)$?

$$f(x+2) = (x+2)^2 - 2(x+2) + 7$$

$$= (x+2)(x+2) - 2x - 4 + 7$$

$$= x^2 + 4x + 4 - 2x - 4 + 7$$

$$\boxed{f(x+2) = x^2 + 2x + 7}$$

Evaluate $f(-x)$?

$$f(-x) = (-x)^2 - 2(-x) + 7$$

$$\boxed{f(-x) = x^2 + 2x + 7.}$$