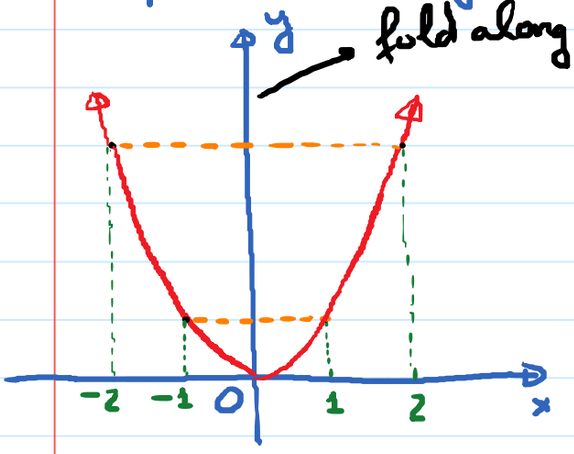
# 2.2. More on Functions and their Graphs.

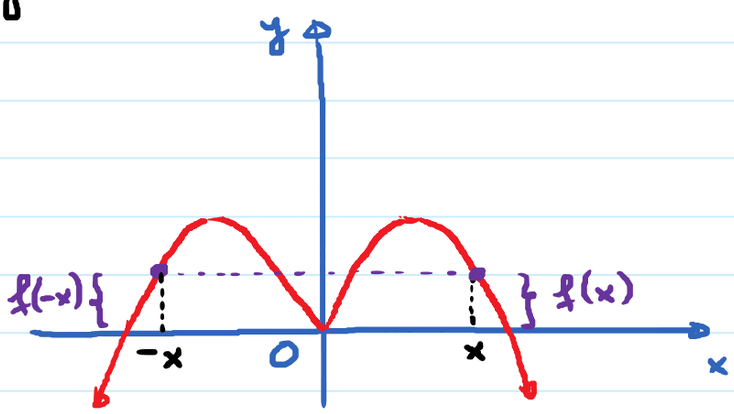
Monday, September 16, 2019 12:50 PM

## Objective 1: Even Functions and Odd Functions

**Definition:** A function whose graph is symmetric with respect to the y-axis is called an EVEN function.




Even function

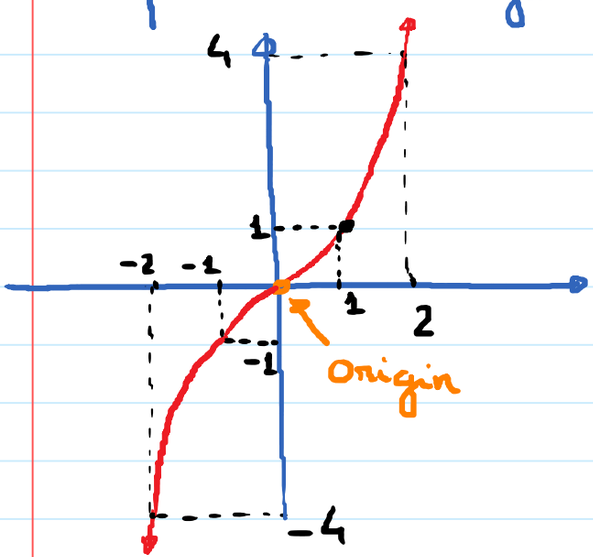



Even function

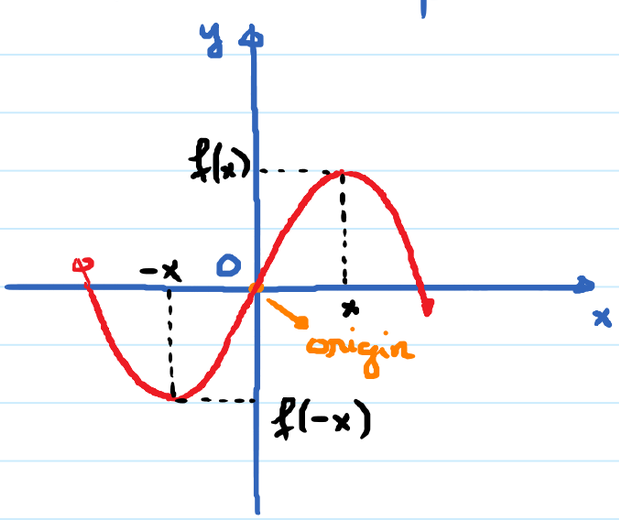
**Note:** If $y = f(x)$ is an even function, then

$$f(-x) = f(x)$$

**Definition:** A function whose graph is symmetric with respect to the origin is called an ODD function




Odd function




Odd function

**Note:** If a function $y = f(x)$ is odd, then:

$$\boxed{f(-x) = -f(x)}$$

* How to determine whether a function $y = f(x)$ is odd or even (or neither) given the equation for $f(x)$ ?

**Step 1:** Find $f(-x)$ by replacing $x$ by $-x$ in the equation for $f(x)$

**Step 2:** Simplify and compare the equation for $f(x)$ and the equation for $f(-x)$.

* If $f(-x) = f(x)$, then $f$ is even
* If $f(-x) = -f(x)$, then $f$ is odd
* If neither, then the function is neither odd nor even.

**E.g.** Determine whether the given function is even or odd or neither.

(a) $\boxed{f(x) = x^4 - 2x^2}$

**Step 1:** Find $f(-x)$: $f(-x) = (-x)^4 - 2(-x)^2$

**Step 2:** Simplify: $\boxed{f(-x) = x^4 - 2x^2}$

$f(-x)$ and $f(x)$ are the same

Compare $f(-x)$ and $f(x)$: $f(-x) = f(x)$

Conclusion: $f$ is an even function.

(b) $f(x) = x^3 - 6x$

Step 1: Find $f(-x)$: $f(-x) = (-x)^3 - 6(-x)$

Step 2: Simplify: $f(-x) = -x^3 + 6x$

Compare $f(x)$ and $f(-x)$:

$f(-x) = -x^3 + 6x = -(x^3 - 6x) = -f(x)$

original $f$

Conclusion: $f$ is an odd function.

(c) $f(x) = x^2 + 2x - 1$

Not Same

Not opposite

$f(-x) = (-x)^2 + 2(-x) - 1$

$= x^2 - 2x - 1$

$f(-x)$

Conclusion: $f$ is neither odd nor even.

## Objective 2: Piecewise Functions.

E.g.

$$f(x) = \begin{cases} 3x + 5 & \text{if } x < 0 \\ 4x + 7 & \text{if } x \geq 0 \end{cases}$$

(a) $f(-2) =$ $-2$ is less than $0$ → use first formula

$f(-2) = 3(-2) + 5 = -6 + 5 = \boxed{-1}$

So, $f(-2) = -1$

(b) $f(3)$ → 3 is greater than 0 → use second formula

$f(3) = 4(3) + 7 = 12 + 7 = 19.$

So, $f(3) = 19.$

(c) $f(0)$ → equal to 0 → use second formula

$f(0) = 4(0) + 7 = 7$

So, $f(0) = 7.$

E.g. Graph the function:

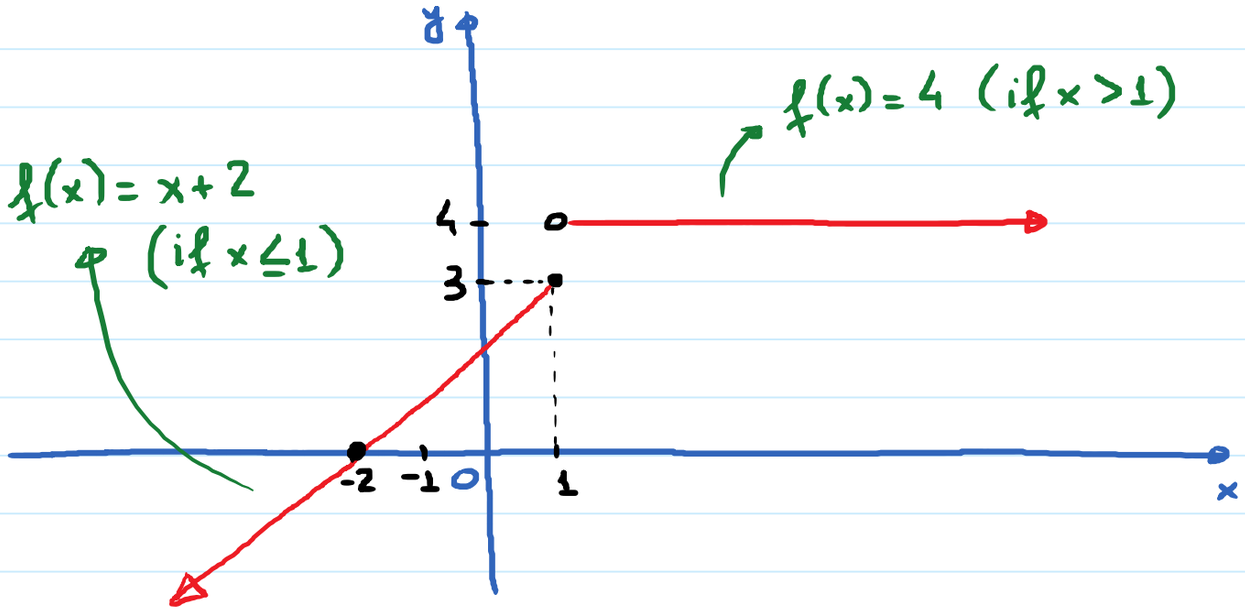
$$f(x) = \begin{cases} x+2 & \text{if } x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$$

Graph each formula:

1st formula: $f(x) = x+2$ (if $x \leq 1$)

| $x$ | $f(x) = x+2$ | ordered pair |
|---|---|---|
| $-2$ | 0 | $(-2, 0)$ |
| 1 | 3 | $(1, 3)$ |

2nd formula: $f(x) = 4$ (if $x > 1$)




**Ex:** Evaluate the function at the given value:

(a) $$g(x) = \begin{cases} x+3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$$

Find $g(0) = 3$; $g(-6) = 3$; $g(-3) = 0$

(b) $$h(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

Find $h(5) = 8$; $h(0) = 3$; $h(3) = 6$