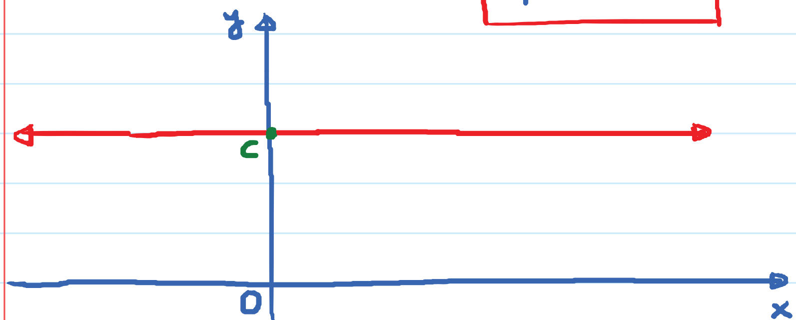
Section 2.5. Transformations of Functions.

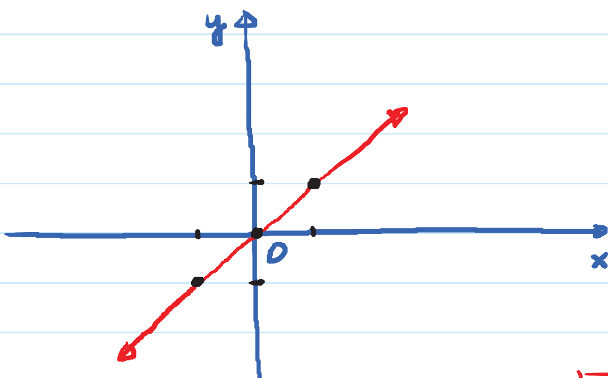
Thursday, September 26, 2019 9:36 AM

Objective 1: Graphs of Common Functions (Parent Functions)

① Constant Function. $\boxed{f(x) = c}$; c is a constant.

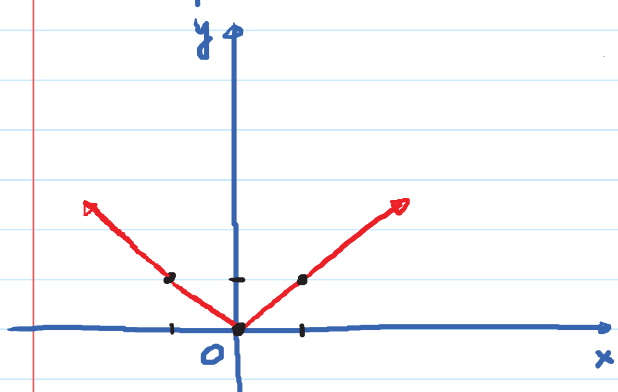


② Identity Function. $\boxed{f(x) = x}$ Graph is a straight line with slope = 1.



③ Absolute Value Function : $\boxed{f(x) = |x|}$

x	$y = f(x) = \lvert x \rvert$
-1	$1 \longrightarrow (-1, 1)$
0	$0 \longrightarrow (0, 0)$
1	$1 \longrightarrow (1, 1)$



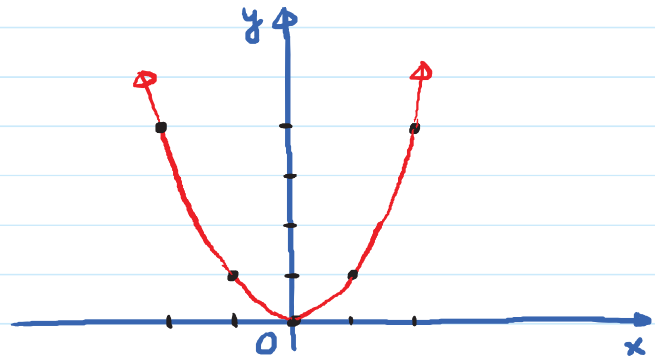
Domain : $(-\infty, \infty)$

Range : $[0, \infty)$

(4) Standard Quadratic Function: $\boxed{f(x) = x^2}$

The graph is a parabola.

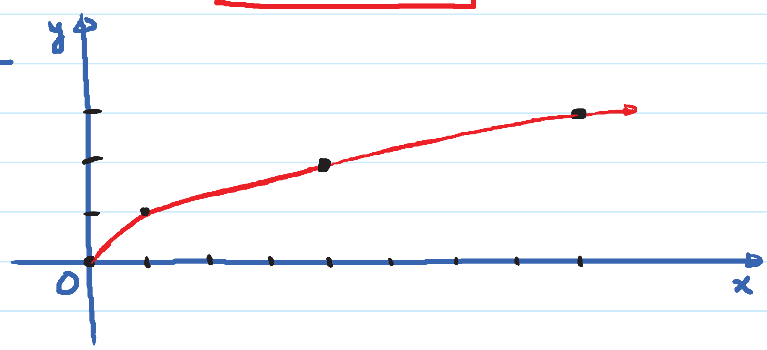
x	$y = f(x) = x^2$
-2	$4 \rightarrow (-2,4)$
-1	$1 \rightarrow (-1,1)$
0	$0 \rightarrow (0,0)$
1	$1 \rightarrow (1,1)$
2	$4 \rightarrow (2,4)$



Domain: $(-\infty,\infty)$; Range: $[0,\infty)$

(5) Square Root Function. $\boxed{f(x) = \sqrt{x}}$

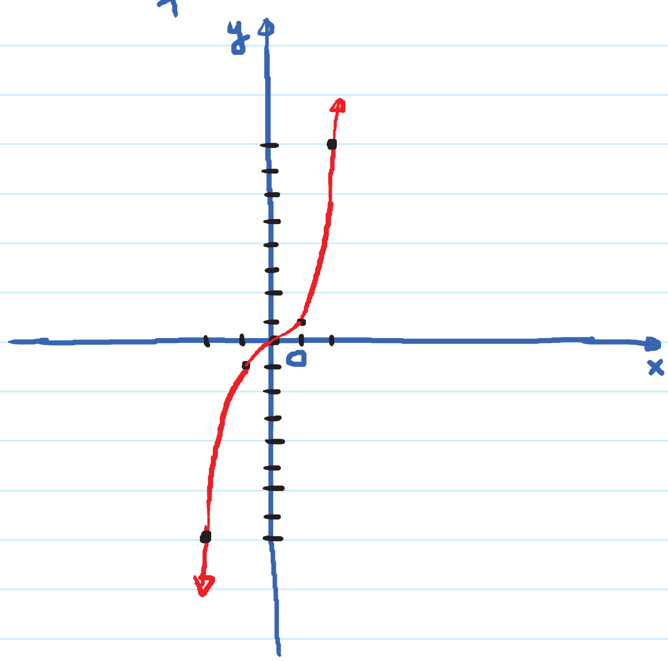
x	$y = f(x) = \sqrt{x}$
0	$0 \rightarrow (0,0)$
1	$1 \rightarrow (1,1)$
4	$2 \rightarrow (4,2)$
9	$3 \rightarrow (9,3)$



Domain: $[0,\infty)$; Range: $[0,\infty)$

(6) Standard Cubic Function. $f(x) = x^3$

x	$y = f(x) = x^3$
-2	$-8 \rightarrow (-2,-8)$
-1	$-1 \rightarrow (-1,-1)$
0	$0 \rightarrow (0,0)$
1	$1 \rightarrow (1,1)$
2	$8 \rightarrow (2,8)$



Domain: $(-\infty,\infty)$

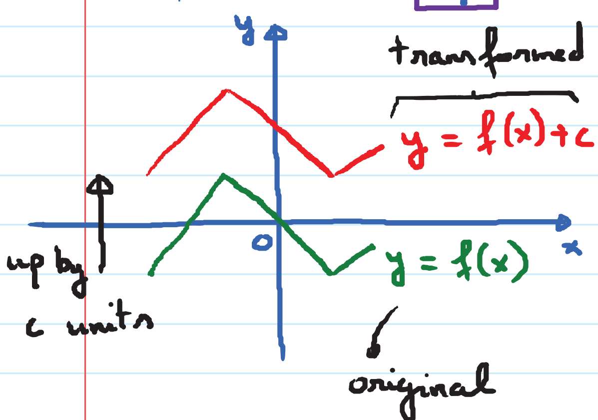
Range: $(-\infty,\infty)$

Objective 2: Vertical Shifts.

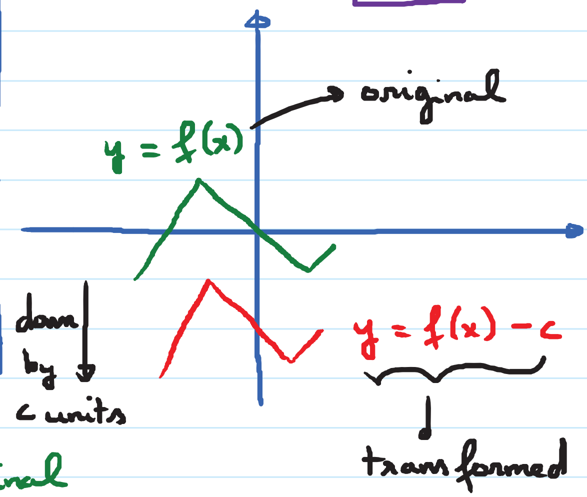
Vertical Shifts:

Given a function $y = f(x)$ and c is a positive constant

The graph of $\boxed{y = f(x) + c}$ is the graph of $\boxed{y = f(x)}$ shifted c units $\boxed{\text{up.}}$



The graph of $\boxed{y = f(x) - c}$ is the graph of $\boxed{y = f(x)}$ shifted c units $\boxed{\text{down}}$



E.g. Given $\boxed{y = f(x) = x^2}$ original

ⓐ What is the formula for $y = f(x) + 2$?

Answer : $y = f(x) + 2 = x^2 + 2$

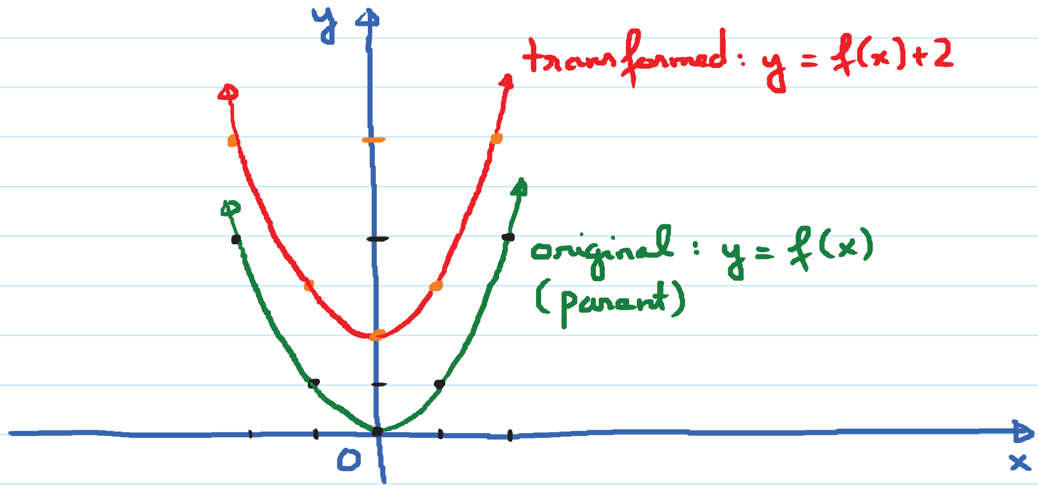
ⓑ Graph both $y = f(x)$ and $y = f(x) + 2$.

x	$y = f(x) = x^2$
-2	$4 \to (-2,4)$
-1	$1 \to (-1,1)$
0	$0 \to (0,0)$
1	$1 \to (1,1)$
2	$4 \to (2,4)$

(original/parent)

x	$y = f(x) + 2 = x^2 + 2$
-2	$6 \to (-2,6)$
-1	$3 \to (-1,3)$
0	$2 \to (0,2)$
1	$3 \to (1,3)$
2	$6 \to (2,6)$

(transformed)

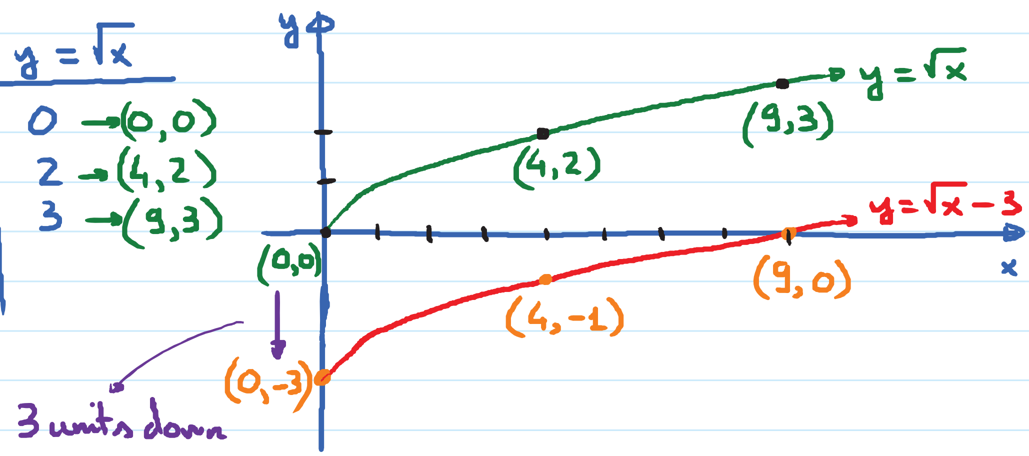


Eg. (a) Graph the square root function $\boxed{y = f(x) = \sqrt{x}}$

(Use 3 Key points).

(b) Use transformation to graph $\boxed{y = \sqrt{x} - 3.}$

x	$y = \sqrt{x}$
0	0 → (0,0)
4	2 → (4,2)
9	3 → (9,3)



Objective 3: Horizontal Shifts.

Horizontal Shifts.

Given a function $y = f(x)$ and a positive number c.

The graph of

$$\boxed{y = f(x+c)}$$

is the graph of

$$\boxed{y = f(x)}$$

shifted $\boxed{\text{left}}$ c units.

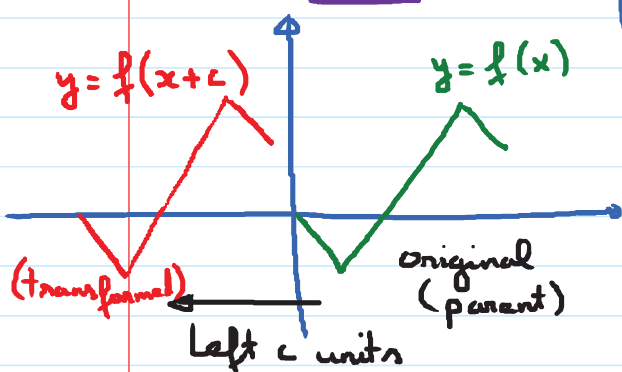
The graph of

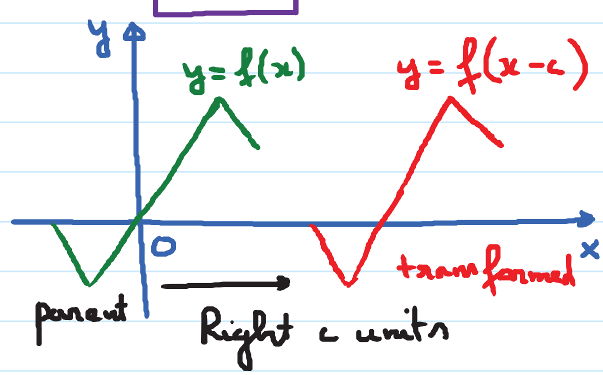
$$\boxed{y = f(x-c)}$$

is the graph of

$$y = f(x)$$

shifted $\boxed{\text{right}}$ c units.





E.g. Given the function $y = f(x) = x^2$.

(a) What is the formula for $y = f(x+2)$?

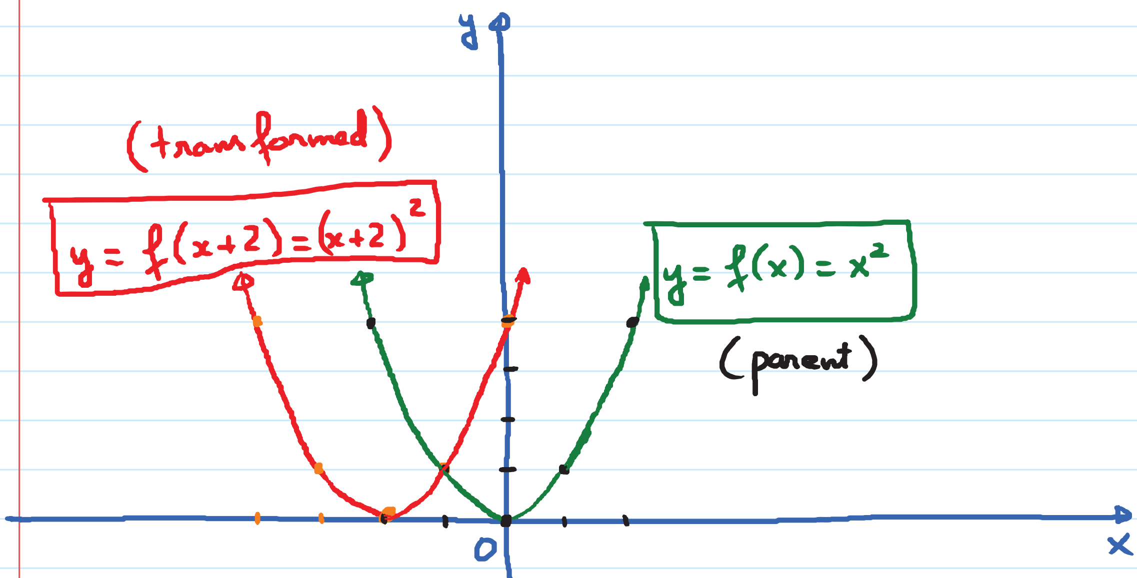
$$y = f(x+2) = (x+2)^2$$

(b) Graph both $y = f(x)$ and $y = f(x+2)$.

x	$y = f(x) = x^2$
-2	$4 \to (-2,4)$
-1	$1 \to (-1,1)$
0	$0 \to (0,0)$
1	$1 \to (1,1)$
2	$4 \to (2,4)$

x	$y = f(x+2) = (x+2)^2$
-4	$(-4+2)^2 = (-2)^2 = 4 \to (-4,4)$
-3	$(-3+2)^2 = (-1)^2 = 1 \to (-3,1)$
-2	$(-2+2)^2 = 0 \to (-2,0)$
-1	$(-1+2)^2 = 1 \to (-1,1)$
0	$(0+2)^2 = 4 \to (0,4)$

left 2 units.



E.g. ⓐ Graph the function $y = f(x) = |x|$. (3 key points)

ⓑ Use transformation to graph $y = f(x-4) = |x-4|$.

x	$y = \lvert x \rvert$	
-1	$1 \to (-1,1)$	$\longrightarrow (3,1)$
0	$0 \to (0,0)$	$\longrightarrow (4,0)$
1	$1 \to (1,1)$	$\longrightarrow (5,1)$

Right 4 units