Section 2.6. Combinations of Functions; Composite Functions

Thursday, October 3, 2019 9:32 AM

Objective 1: Find a Function's Domain.

Note: The domain of a function is the set of all real numbers: $(-\infty, \infty)$ unless x appears in the denominator or x appears in a square root (or an even root).

* **Case 1:** Functions that do not have x appears in the denominator or in a square root.

E.g. Find the domain of $f(x) = x^2 - 7x$.

Answer: Since the function contains neither a denominator nor a square root, the domain is the set of all real numbers.

In interval notation: $\boxed{D_f = (-\infty, \infty)}$

* **Case 2:** Functions that have x appear in the denominator.

Strategy for finding the domain of these functions:

Step 1: Set denominator equal to zero and solve for x.

Step 2: The domain is the set of all real numbers excluding the values in Step 1. Write in interval notation.

E.g. Find the domain of the given function

(a) $f(x) = \dfrac{1}{x-6}$ (b) $g(x) = \dfrac{2x+3}{12x-7}$

(c) $h(x) = \dfrac{3x+2}{x^2-2x-3}$ (d) $j(x) = \dfrac{5x}{x^2-49}$

Solution:

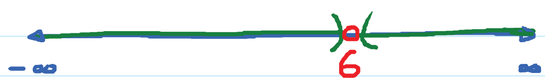
(a) Step 1: $x - 6 = 0$ (Set denom. $= 0$)

$x = 6$

Step 2:

$D_f =$ all real numbers excluding 6

In interval notation:



$$\boxed{D_f = (-\infty, 6) \cup (6, \infty)}$$

union

In set notation:

$$\boxed{D_f = \{x \mid x \neq 6\}}$$

(b) $g(x) = \dfrac{2x+3}{\boxed{12x-7}}$ → Denominator

Step 1: $12x - 7 = 0$ (Set Denom. $= 0$)

$x = \dfrac{7}{12}$

Step 2: $\boxed{D_g = \left(-\infty, \dfrac{7}{12}\right) \cup \left(\dfrac{7}{12}, \infty\right)}$

(c) $h(x) = \dfrac{3x+2}{\boxed{x^2 - 2x - 3}}$ → denominator

Step 1: $x^2 - 2x - 3 = 0$ (Set denom. $= 0$)

$(x-3)(x+1) = 0$ (Factor)

$x - 3 = 0$	$x + 1 = 0$
$x = 3$	$x = -1$

Step 2: D_h = all real numbers except for -1 and 3.

In interval notation: 

$$\boxed{D_h = (-\infty, -1) \cup (-1, 3) \cup (3, \infty)}$$

In set notation:

$$\boxed{D_h = \{x \mid x \neq -1,\ x \neq 3\}}$$

(d) $j(x) = \dfrac{5x}{x^2 - 49}$

Step 1: $x^2 - 49 = 0$ (Set Denom. $= 0$)

$\rightarrow x^2 = 49 \rightarrow x = \pm\sqrt{49} = \pm 7$

Step 2: D_j = all real numbers except for -7 and 7

$$\boxed{D_j = (-\infty, -7) \cup (-7, 7) \cup (7, \infty)}$$

$$\boxed{D_j = \{x \mid x \neq -7,\ x \neq 7\}}$$

Case 3: Functions that have x appear in a square root

Strategy for finding the domain of these functions

Step 1: Set the expression under the square root to be ≥ 0.

We have an inequality. Solve this inequality.

Step 2: The solution to the inequality is the domain.

Write the domain in interval notation.

E.g. Find the domain of the given function.

(a) $f(x) = \sqrt{3x + 12}$

(b) $g(x) = \sqrt{27 - 9x}$

Solution:

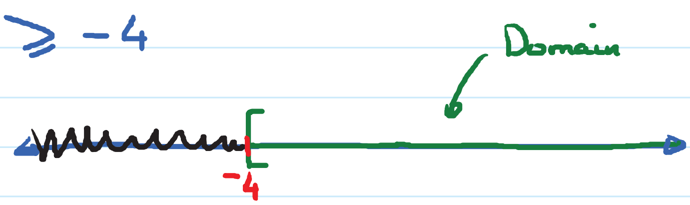
(a) **Step 1:** $3x + 12 \geq 0$ (Set expr. under $\sqrt{} \geq 0$)

$\rightarrow 3x \geq -12$ (Subtract 12)

$\rightarrow x \geq \dfrac{-12}{3}$ (Divide by 3)

$\rightarrow x \geq -4$

Step 2:



$\boxed{D_f = [-4, \infty)}$; $\boxed{D_f = \{x \mid x \geq -4\}}$

(interval) (set)

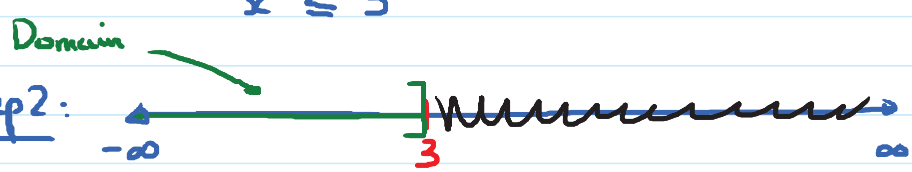
(b) $g(x) = \sqrt{27 - 9x}$

Step 1: $27 - 9x \geq 0$

$-9x \geq -27$

$x \leq \dfrac{-27}{-9}$ (when we divide by a negative # in an inequality, the direction is changed)

$x \leq 3$

Step 2: 

$$\boxed{D_g = (-\infty, 3]}$$

Case 4: Functions that have x appear in a square root that is in the denominator.

Strategy for finding the domain:

Step 1: Set the expression under the square root to be > 0.

Solve the resulting inequality.

Step 2: The domain is the solution the inequality.

Write it in interval notation.

E.g. Find the domain

(a) $f(x) = \dfrac{3x+2}{\sqrt{2x-20}}$

(b) $g(x) = \dfrac{5x}{\sqrt{24-3x}}$

Solution:

(a) Step 1: $2x - 20 > 0$

$\rightarrow 2x > 20 \rightarrow x > 10$

Step 2: $\boxed{D_f = (10, \infty)}$

(b) Step 1: $24 - 3x > 0$

$\rightarrow -3x > -24 \rightarrow x < \dfrac{-24}{-3}$

$\rightarrow x < 8$

Step 2: $\boxed{D_g = (-\infty, 8)}$

Obj 2: Combining Functions.

E.g. Given 2 function $f(x) = 2x$; $g(x) = x - 1$

(1) Sum $f + g$

$$\boxed{(f+g)(x) = f(x) + g(x)}$$

$$(f+g)(x) = (\underbrace{2x}_{f(x)}) + (\underbrace{x-1}_{g(x)}) = 3x - 1$$

plug 5 into formula for $f+g$

$$(f+g)(5) \to \underbrace{3(5)-1}_{\text{plug 5 into formula for } f+g} = 14$$

or

$$\underbrace{f(5)}_{2(5)} + \underbrace{g(5)}_{5-1} = 10+4 = 14$$

(2) Difference: $f-g$

$$\boxed{(f-g)(x) = f(x) - g(x)}$$

$$(f-g)(x) = (\underbrace{2x}_{f(x)}) - (\underbrace{x-1}_{g(x)})$$

$$= 2x - x + 1 = x+1$$

(3) Product: fg

multiply

$$\boxed{(fg)(x) = f(x)\cdot g(x)}$$

$$(fg)(x) = (\underbrace{2x}_{f(x)})\cdot(\underbrace{x-1}_{g(x)}) = 2x^2 - 2x$$

$$(fg)(5) \to 2(5)^2 - 2(5) = 50 - 10 = 40$$

or

$$\underbrace{(2\cdot 5)}_{f(5)}\cdot\underbrace{(5-1)}_{g(5)} = 10\cdot 4 = 40$$

(4) Quotient: $\frac{f}{g}$

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}}$$

$$\left(\frac{f}{g}\right)(x) = \frac{2x}{x-1}$$

$$\left(\frac{f}{g}\right)(5) \to \frac{2(5)}{5-1} = \frac{10}{4} = \frac{5}{2}$$

or

$$\to \frac{f(5)}{g(5)} = \frac{2(5)}{5-1} = \frac{10}{4} = \frac{5}{2}$$

E.g. $f(x) = 2x-1$. $g(x) = x^2+x-2$.

(1) Find formulas for the functions

(a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $(fg)(x)$ (d) $\left(\frac{f}{g}\right)(x)$

(2) Find the domain of $f+g$; $f-g$; fg and $\frac{f}{g}$

Solution:

(1) (a) $(f+g)(x) = f(x) + g(x)$

$$= (2x-1) + (x^2+x-2)$$

$$\boxed{(f+g)(x) = x^2+3x-3}$$ (Combine like terms)

(b) $(f-g)(x) = f(x) - g(x)$

$$= (2x-1) - (x^2+x-2)$$

$$= 2x-1-x^2-x+2$$ (Distribute the $-$)

$$\boxed{(f-g)(x) = -x^2+x+1}$$ (Combine like terms)