(c) $(fg)(x) = f(x) \cdot g(x)$

$= (2x - 1) \cdot (x^2 + x - 2)$ (Distribute)

$= 2x^3 + 2x^2 - 4x - x^2 - x + 2$

$\boxed{(fg)(x) = 2x^3 + x^2 - 5x + 2}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{2x - 1}{x^2 + x - 2} = \frac{2x - 1}{(x + 2)(x - 1)}}$

(2) Find the domain of $f+g$, $f-g$, fg and $\frac{f}{g}$

* The domains of $f+g$, $f-g$, fg are the same.

We use the same process.

Step 1: Find the domain of f and domain g.

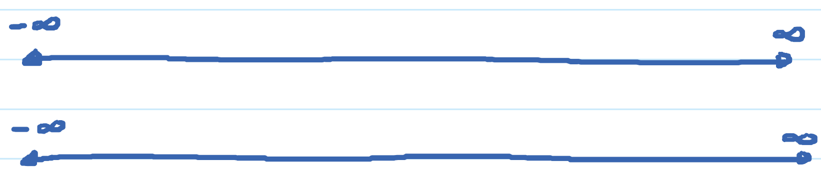
$f(x) = 2x - 1$. $D_f = (-\infty, \infty)$ (No restrictions)

$g(x) = x^2 + x - 2$. $D_g = (-\infty, \infty)$ (No restrictions)

Step 2: Find the intersection $D_f \cap D_g$ intersection

D_f :

D_g :



So, $D_f \cap D_g = (-\infty, \infty)$

Conclusion: $\boxed{\text{Domain of } f+g,\ f-g,\ fg \text{ is } (-\infty, \infty)}$

* To find the domain of $\frac{f}{g}$, we need an additional step.

<u>Step 1</u>: (Same as previous): Find D_f and D_g

<u>Step 2</u>: (Same as previous): Find $D_f \cap D_g$.

We found this: $D_f \cap D_g = (-\infty, \infty)$

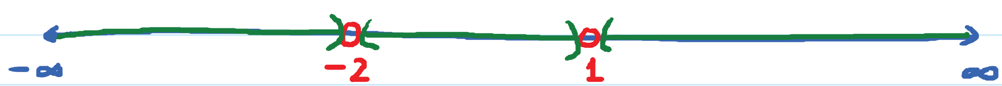
<u>Step 3</u>: Set $g(x) = 0$. Solve for x and exclude these values from the domain in Step 2.

$$g(x) = 0 \longrightarrow x^2 + x - 2 = 0$$

$$\rightarrow (x+2)(x-1) = 0$$

$$\rightarrow x + 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\rightarrow x = -2 \quad \text{or} \quad x = 1$$



$$\boxed{\text{Domain of } \frac{f}{g} = (-\infty,-2) \cup (-2,1) \cup (1,\infty)}$$

<u>E.g.</u> Given $f(x) = \sqrt{x-3}$ and $g(x) = \sqrt{x+1}$

1. Find $(f+g)(x)$
2. Find $(f+g)(3)$
3. Find Domain of $f+g$.

<u>Solution</u>: (1) $(f+g)(x) = f(x) + g(x)$

$$\boxed{(f+g)(x) = \sqrt{x-3} + \sqrt{x+1}}$$

(2) $(f+g)(3) = \sqrt{3-3} + \sqrt{3+1}$

$= \sqrt{0} + \sqrt{4} = 0 + 2 = 2$

(3) Domain of $f+g$

Step 1: Find domain of f and domain of g

$f(x) = \sqrt{x-3}$. Set $x - 3 \geq 0$

$\rightarrow x \geq 3$

D_f

$g(x) = \sqrt{x+1}$. Set $x + 1 \geq 0$

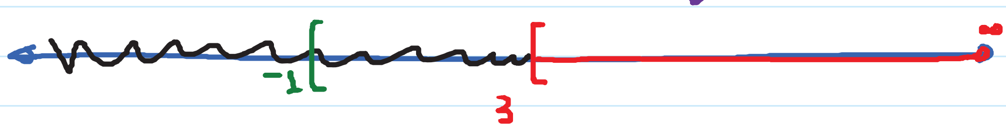
$\rightarrow x \geq -1$

D_g

Step 2: Find $D_f \cap D_g$

Intersection

$D_f \cap D_g$



Conclusion: Domain of $f+g$ is : $\boxed{[3, \infty)}$

Obj 3: Composite functions (Composition of functions)

Given functions f and g. The composition of f with g is a function. This function is denoted by $f \circ g$ (read as f circle g or f of g). This function is defined by the formula:

$$\boxed{(f \circ g)(x) = f(\underbrace{g(x)}_{\text{plug } g(x) \text{ into } f})}$$

E.g. Given : $f(x) = 3x - 4$

$g(x) = x^2 - 2x + 6.$

Find : ① $(f \circ g)(x)$ ② $(g \circ f)(x)$

③ $(f \circ g)(1)$ ④ $(g \circ f)(1)$

Solution:

① $(f \circ g)(x) = f(g(x))$

$f(x) = 3\boxed{x} - 4$

$g(x) = \boxed{x^2 - 2x + 6}$

So, $f(g(x)) = 3(x^2 - 2x + 6) - 4$

$= 3x^2 - 6x + 18 - 4$

$$\boxed{(f \circ g)(x) = 3x^2 - 6x + 14}$$

② $(g \circ f)(x) = g(f(x))$ (Plug $f(x)$ into g)

$g(x) = \boxed{x}^2 - 2\boxed{x} + 6$

$f(x) = \boxed{3x - 4}$

$$g(f(x)) = (3x-4)^2 - 2(3x-4) + 6$$

$$= (3x-4)(3x-4) - 6x + 8 + 6$$

$$= 9x^2 - 12x - 12x + 16 - 6x + 14$$

$$\boxed{g(f(x)) = 9x^2 - 30x + 30}$$

c) $(f \circ g)(1)$

<u>Method 1:</u> Plug $x=1$ into the formula for $f \circ g$ which we found in part a)

$$(f \circ g)(x) = 3x^2 - 6x + 14$$

$$(f \circ g)(1) = 3(1)^2 - 6(1) + 14$$

$$= 3 - 6 + 14 = \boxed{11}$$

<u>Method 2:</u> $(f \circ g)(1) = f(g(1))$

Plug 1 into g and then plug what we get into f.

$$f(x) = 3x - 4$$

$$g(x) = x^2 - 2x + 6$$

$$\longrightarrow g(1) = (1)^2 - 2(1) + 6 = 1 - 2 + 6 = 5$$

$$\longrightarrow f(g(1)) = f(5) = 3(5) - 4 = \boxed{11}$$

d) $(g \circ f)(1)$

<u>Method 1:</u> $(g \circ f)(x) = 9x^2 - 30x + 30$

$$(g\circ f)(1) = 9(1)^2 - 30(1) + 30$$

$$= 9 - 30 + 30 = \boxed{9}$$

Method 2: Plug 1 into f and plug what we got into g

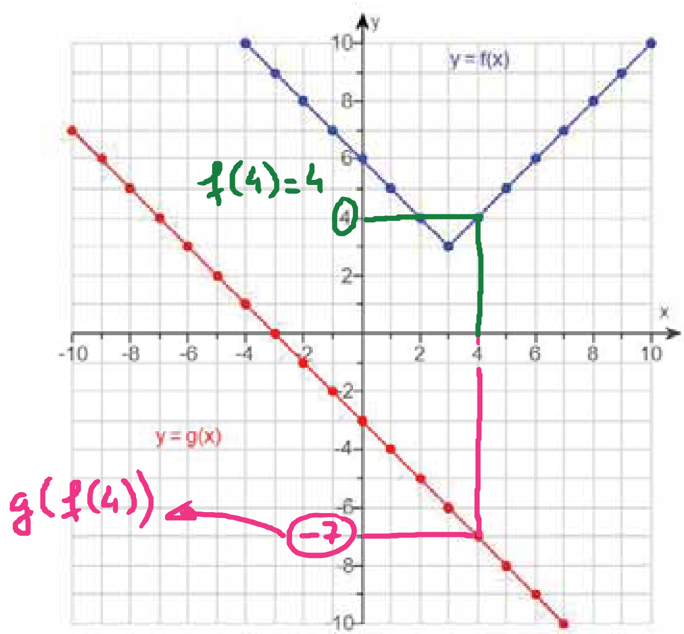
$$f(x) = 3x-4 \longrightarrow f(1) = 3(1)-4 = \boxed{-1}$$

$$g(x) = x^2 - 2x + 6$$

$$g(f(1)) = g(-1) = (-1)^2 - 2(-1) + 6$$

$$= 1 + 2 + 6 = \boxed{9}.$$

E.g.



Find $g(f(4))$

Ans: $\boxed{-7}$