Section 3.1. Quadratic Functions

Tuesday, October 22, 2019 9:35 AM

Obj 1: Characteristics of Parabolas.

A quadratic function is a function of the form

$$\boxed{f(x) = ax^2 + bx + c}$$

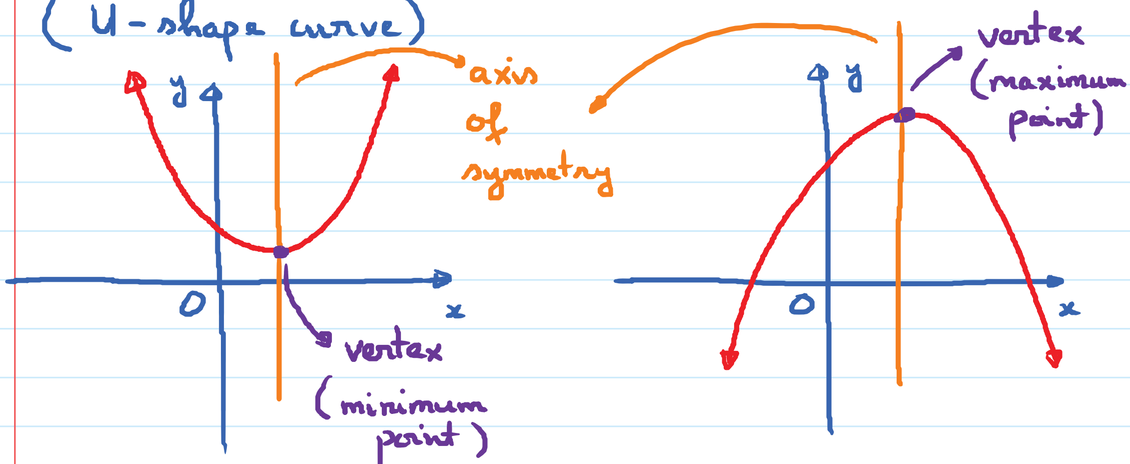
where a, b, c are real numbers and $a \neq 0$

E.g. $f(x) = 2x^2 + 3x - 5$. $(a = 2;\ b = 3;\ c = -5)$

$g(x) = -3x^2 + 4x - 7$. $(a = -3;\ b = 4;\ c = -7)$

The graph of any quadratic function is a parabola

(U-shape curve)



opens upward if $a > 0$

opens downward if $a < 0$

Obj 2: Graph Quadratic Functions $f(x) = ax^2 + bx + c$

* Vertex Formula.

Given a quadratic function $f(x) = ax^2 + bx + c$.

$$\boxed{x_{\text{vertex}} = -\frac{b}{2a}\ ;\ y_{\text{vertex}} = f\left(-\frac{b}{2a}\right)}$$

Note: To find y_{vertex}, we plug the value of x_{vertex} into the function.

E.g. Find the vertex of the parabola defined by the given function.

(a) $f(x) = 2x^2 - 8x + 3$

(b) $f(x) = -x^2 - 2x + 8$

Solution:

(a) $a = 2$; $b = -8$; $c = 3$

$$x_{vertex} = -\frac{b}{2a} = -\frac{(-8)}{2(2)} = 2$$

$$y_{vertex} = 2(2)^2 - 8(2) + 3 = -5$$

Vertex : $(2, -5)$

(b) $a = -1$; $b = -2$; $c = 8$

$$x_{vertex} = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$$

$$y_{vertex} = -(-1)^2 - 2(-1) + 8 = 9$$

Vertex : $(-1, 9)$

* Process to graph a quadratic function

$f(x) = ax^2 + bx + c$

Step 1: Determine whether the parabola opens up or opens down.

$a > 0 \rightarrow$ opens up ; $a < 0 \rightarrow$ opens down.

Step 2: Find the vertex.

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Step 3: Find the axis of symmetry.

$$x = -\frac{b}{2a}$$

Step 4: Find the y-intercept.

Set $x = 0$ in the function $f(x) = ax^2 + bx + c$.

We obtain c.

So, the y-intercept is $(0, c)$

We can reflect the y-intercept across the axis of symmetry to get an additional point.

Step 5: Find the x-intercept(s)

Set $f(x) = 0$. We have a quadratic equation

$$ax^2 + bx + c = 0$$

The solutions (if real) are the x-coordinates of the x-intercepts.

Step 6: Generate additional points if necessary by using a T-table

x	y

E.g. Graph $f(x) = x^2 - 2x - 3$.

Step 1: $a = 1 > 0$. Parabola opens up.

Step 2: $x_{\text{vertex}} = -\frac{(-2)}{2(1)} = 1$

$y_{\text{vertex}} = (1)^2 - 2(1) - 3 = -4$

Vertex $(1, -4)$

Step 3: Vertical line $x = 1$

Step 4: y-intercept : $(0, c) = (0, -3)$

Step 5: x-intercepts : $x^2 - 2x - 3 = 0$

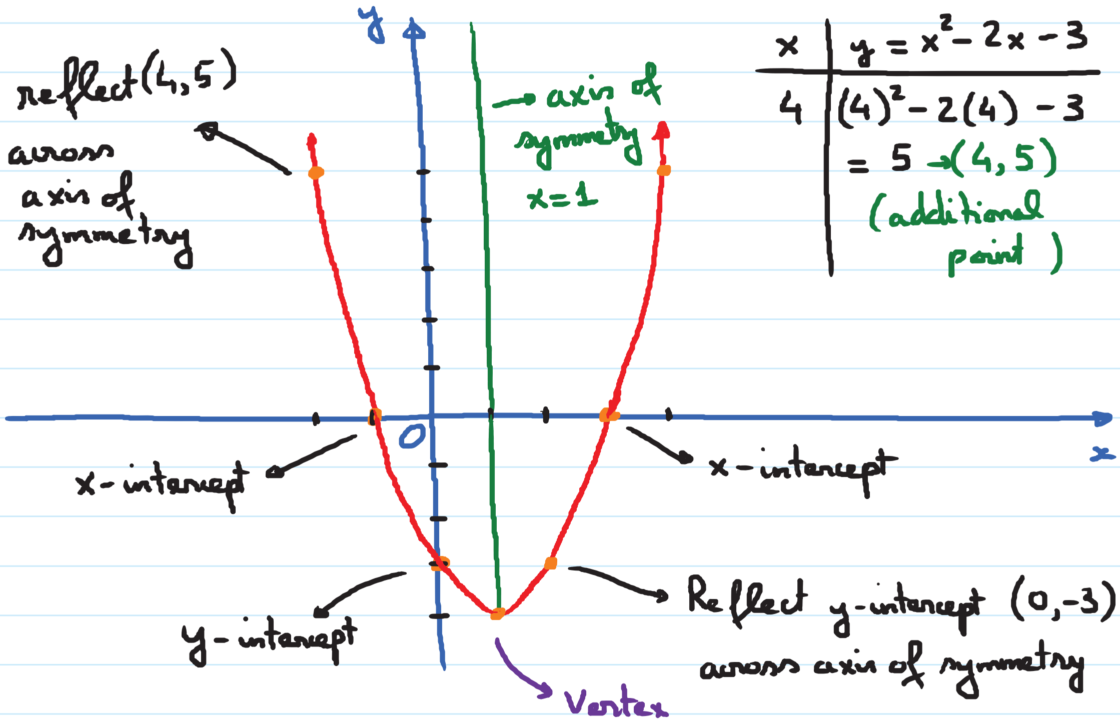
$$(x+1)(x-3) = 0$$

$x + 1 = 0$; $x - 3 = 0$

$x = -1$ $\quad$ $x = 3$

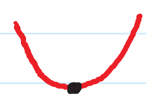
x-intercepts : $(-1, 0)$; $(3, 0)$

x	$y = x^2 - 2x - 3$
4	$(4)^2 - 2(4) - 3$ $= 5 \rightarrow (4, 5)$ (additional point)

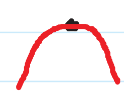


Obj 3: Find maximum / minimum values of quadratic functions: $f(x) = ax^2 + bx + c$

Process:

If $a > 0$, f has a minimum point. 

The minimum value $= y_{\text{vertex}}$ and it occurs when $x = x_{\text{vertex}}$.

If $a < 0$, f has a maximum point. 

The maximum value $= y_{\text{vertex}}$ and it occurs when $x = x_{\text{vertex}}$.

E.g. Given $f(x) = -3x^2 + 6x - 13$.

Q1: Determine whether f has a maximum or a minimum value.

Q2: Find the max / min value and determine where it occurs.

Q3: Find the domain and range of this function.

Solution:

Q1: Since $a = -3 < 0$, parabola opens down.

So, f has a maximum value.

Q2: Find vertex:

$$x_{vertex} = -\frac{b}{2a} = -\frac{6}{2(-3)} = 1$$

$$y_{vertex} = -3(1)^2 + 6(1) - 13 = -10.$$

Conclusion: Maximum value $= -10$

and it occurs when $x = 1$.

Q3: Domain of $f = (-\infty, \infty)$

(Reason: function has no denominator with x or square root with x)

Range $= (-\infty, -10]$

Note: If $a > 0$, then the range of

$f(x) = ax^2 + bx + c$ is : $[y_{\text{vertex}}, \infty)$

If $a < 0$, then the range of

$f(x) = ax^2 + bx + c$ is : $(-\infty, y_{\text{vertex}}]$