$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

Solution set: $\boxed{\left\{ 2 \,;\, \frac{-3+\sqrt{5}}{2} \,;\, \frac{-3-\sqrt{5}}{2} \right\}}$

<u>Obj 3:</u> Find a polynomial function with given zeros.

Properties of Zeros (Roots) of polynomial functions.

1. If a polynomial function has degree n, then it has n zeros (roots) counting multiplicity

2. For imaginary zeros (roots):

Imaginary zeros always appear in conjugate pairs; i.e, if $a + bi$ is a zero of f, then $a - bi$ is also a zero.

<u>E.g.</u> If $f(x)$ has $2+3i$ as a zero, then $2-3i$ is also a zero.

3. If c is a zero (root) of $f(x)$, then $x - c$ is a factor of $f(x)$.

<u>E.g.</u> If 3 is a zero of f, then $x-3$ is a factor of f.

If -7 is a zero of f, then $x+7$ is a factor of f

4 If $c_1, c_2, \dots, c_n$ are all the zeros of $f(x)$, then $f(x)$ can be written in factored form as:

leading coefficient.

$$f(x) = \boxed{a}(x-c_1)(x-c_2)\cdots(x-c_n)$$

---

* Find polynomial functions with given zeros.

E.g. Find a degree 4 polynomial function given:

a) $-2, 2, i$ are zeros of f.

b) $f(3) = -150$.

Sol:

Since degree $= 4$, there must be 4 zeros. We are given 3 zeros. By the 2nd property above, $-i$ must also be a zero.

All the zeros of f are: $-2, 2, i, -i$

By property 4, f can be written in factored form as

$$f(x) = a\boxed{(x+2)(x-2)}\boxed{(x-i)(x+i)}$$

Now, we expand and simplify.

-1

$$f(x) = a(x^2-4)(x^2-\boxed{i^2})$$

$$f(x) = a(x^2-4)(x^2+1)$$

$$f(x) = a\cdot(x^4+x^2-4x^2-4)$$

$$f(x) = a(x^4-3x^2-4)$$

Since $f(3) = -150$, we have:

$$a(\underbrace{3^4-3\cdot 3^2-4}) = -150$$

$$a\cdot(81-27-4) = -150$$

$$a\cdot(50) = -150$$

$$\rightarrow a = \frac{-150}{50} = -3$$

$$f(x) = -3(x^4-3x^2-4)$$

$$\boxed{f(x) = -3x^4+9x^2+12}$$

E.g. Find a third-degree polynomial function $f(x)$ given:

1. -3 and i are zeros.
2. $f(1) = 8$

Since $f(x)$ has degree 3, it has 3 zeros.

Since imaginary zeros occur in conjugate pair and i is a zero, $-i$ is also a zero.

So, the zeros of $f(x)$ are: $-3, i, -i$

We can write $f(x)$ in factored form as:

leading coeff

$$f(x) = \boxed{a}(x+3)(x-i)(x+i)$$

-1

$$= a(x+3)(x^2 - \boxed{i^2})$$

$$= a(x+3)(x^2 - (-1))$$

$$= a(x+3)(x^2+1)$$

$$= a(x^3 + 3x^2 + x + 3)$$

Since $f(1) = 8$, we have:

$$8 = a \cdot (1^3 + 3(1)^2 + 1 + 3)$$

$$8 = a \cdot 8 \longrightarrow a = \frac{8}{8} = 1$$

So, $\boxed{f(x) = x^3 + 3x^2 + x + 3}$