3.5. Rational Functions and their graphs.

Tuesday, November 5, 2019 9:45 AM

A rational function is a function of the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials.

E.g.

$$f(x) = \frac{x^2+1}{3x^3+7x^2-5x+12}$$

$$f(x) = \frac{x^2+7x+9}{x(x-2)(x+5)}$$

rational functions

Obj 1: Find the domain of a rational function.

Process:

Step 1: Set denominator $= 0$ and solve for x.

Step 2: Domain = set of all real numbers except for the values you found in Step 1.

E.g. Find the domain of each rational function

(a) $f(x) = \dfrac{x^2-25}{x-5}$ (b) $g(x) = \dfrac{x}{x^2-25}$

(c) $h(x) = \dfrac{x+7}{x^2+4x-21}$ (d) $w(x) = \dfrac{x+5}{x^2+25}$

Solution

(a) Set $x-5=0$; $x = 5$. (Set denom $=0$ and solve)

$\text{Domain} = \{x \mid x \neq 5\}$

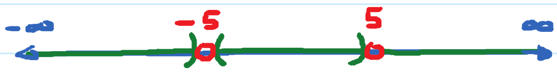
or in interval notation: 

$\text{Domain} = (-\infty, 5) \cup (5, \infty)$

(b) Set $x^2 - 25 = 0 \longrightarrow x^2 = 25$

$\longrightarrow x = \pm\sqrt{25} = \pm 5$

$\text{Domain} = \{x \mid x \neq 5;\ x \neq -5\}$

or in interval notation: 

$\text{Domain} = (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

(c) Set $x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$x + 7 = 0 \quad ; \quad x - 3 = 0$

$x = -7 \qquad\qquad x = 3$

$D = \{x \mid x \neq -7,\ x \neq 3\}$

or $D = (-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

(d) Set $x^2 + 25 = 0 \longrightarrow x^2 = -25$

$x = \pm\sqrt{-25} \longrightarrow x = \pm i \cdot 5$

(imaginary numbers)

So, the domain is the set of all real numbers

$D = (-\infty, \infty)$

Obj 2: Find vertical asymptotes of Rational Functions

E.g. $f(x) = \dfrac{x+7}{x^2+4x-21}$

Step 1: Factor top and bottom completely

$$f(x) = \frac{\cancel{x+7}}{\cancel{(x+7)}(x-3)}$$

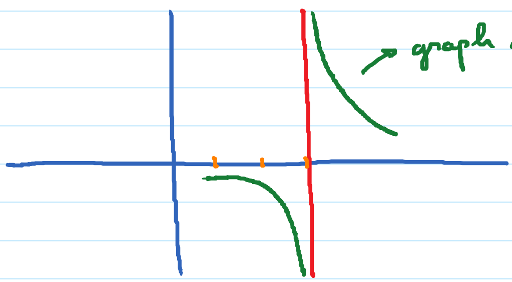
Step 2: Cancel the common factor(s) if any

$$= \boxed{\frac{1}{x-3}}$$

Step 3: Set denominator of $\boxed{\text{the simplified expression}}$ equal to zero and solve.

$$x-3=0 \rightarrow x=3.$$

The vertical asymptote is $x=3$.

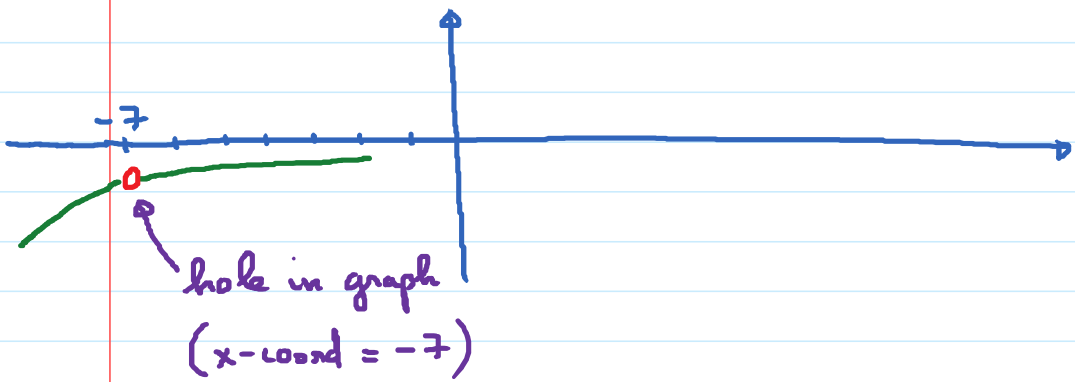


graph approaches $x=3$ but does not touch that vertical line.

vertical asymptote $x=3$

Note: Set the canceled factor $=0$ to solve for the x-coordinate of the hole in the graph.

Canceled factor $x+7=0 \rightarrow x=-7$



E.g. Find the vertical asymptote(s) if any of the function.

(a) $f(x) = \dfrac{x}{x+4}$ (b) $f(x) = \dfrac{x^2-25}{x-5}$

(c) $f(x) = \dfrac{x-5}{x^2-25}$ (d) $f(x) = \dfrac{x^2}{x^2+x-6}$

Sol:

(a) $f(x) = \dfrac{x}{x+4}$. Set $x+4=0 \rightarrow \boxed{x=-4}$ V.A.

(b) $f(x) = \dfrac{\cancel{(x-5)}(x+5)}{\cancel{x-5}}$ (Factor)

$\dfrac{x+5}{1}$ (Cancel)

Set $1 = 0$ (No solution)

Conclusion: No V.A.

(c) $f(x) = \dfrac{x-5}{x^2-25} = \dfrac{\cancel{x-5}}{\cancel{(x-5)}(x+5)}$ (Factor)

$= \dfrac{1}{x+5}$ (cancel)