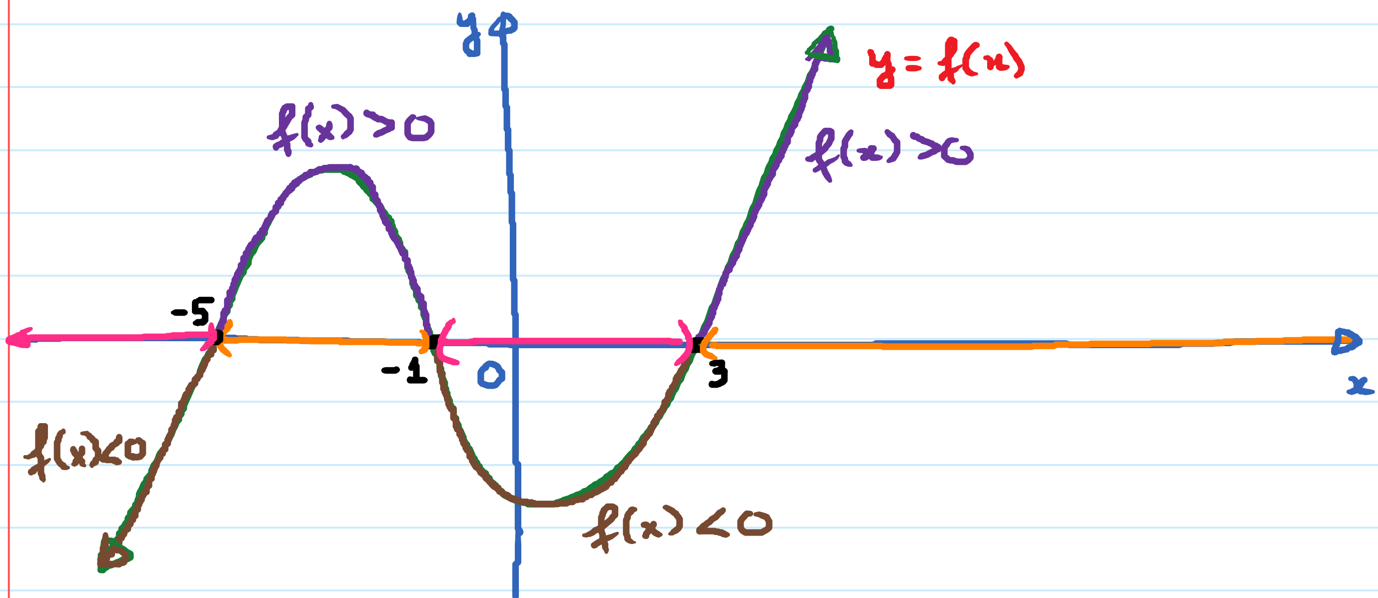
3.6. Polynomial and Rational Inequalities

Thursday, November 14, 2019 9:45 AM

What does it mean to solve an inequality of the form $f(x) > 0$ or $f(x) < 0$ or $f(x) \geq 0$ or $f(x) \leq 0$?

Given the graph of $y = f(x)$:



When we solve the equation $f(x) = 0$, we are looking for the x - intercepts.

So, the solution set to the equation $f(x) = 0$ is

$\{-5, -1, 3\}$

When we solve the inequality $f(x) > 0$, we are looking for the interval(s) over which the of $f(x)$ is above the x-axis $(y>0)$.

Similarly, we we solve $f(x)<0$, we are looking for the interval(s) over which $f(x)$ is below the x-axis $(y<0)$

Inequality	Solution set
$f(x)>0$	$(-5,-1)\cup(3,\infty)$
$f(x)<0$	$(-\infty,-5)\cup(-1,3)$
$f(x)\geq 0$	$[-5,-1]\cup[3,\infty)$
$f(x)\leq 0$	$(-\infty,-5]\cup[-1,3]$

Obj 1: Solve Polynomial Inequalities

E.g. Solve the inequality and graph the solution set on a number line:

$$\boxed{x^2-x>20}$$

Step 1: Express the inequality in the form

$f(x) > 0$ or $f(x) < 0$ or $f(x) \geq 0$ or

$f(x) \leq 0$

$$x^2 - x > 20$$

$$\underbrace{\boxed{x^2 - x - 20}}_{f(x)} > 0$$ (Subtract 20 from both sides)

Step 2: Find the "special" values of x by setting $f(x) = 0$ and solve.

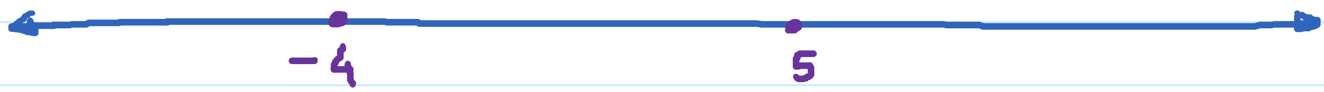
$$x^2 - x - 20 = 0$$ (Set $f(x) = 0$)

$$(x-5)(x+4) = 0$$ (Factor)

$$x - 5 = 0 \quad ; \quad x + 4 = 0$$

$$\underbrace{\boxed{x = 5} \quad ; \quad \boxed{x = -4}}_{\text{"Special" values of } x}$$

Step 3: Locate these special values on a number line and separate the line into intervals



These values divide the line into 3 intervals:

$(-\infty, -4)$; $(-4, 5)$; $(5, \infty)$

Since we are solving the inequality $f(x) > 0$, we just need to determine the interval(s) for which $f(x) > 0$ (positive)

Step 4: Choose one test value within each interval and plug it into $f(x)$ to determine the sign of $f(x)$

Interval	$(-\infty, -4)$	$(-4, 5)$	$(5, \infty)$
Test Value	-5	0	6
Plug it into $f(x) = x^2 - x - 20$	10	-20	10
	$f(x) > 0$	$f(x) < 0$	$f(x) > 0$

Step 5: Write the solution set by selecting the interval(s) that satisfy the inequality.

Since we are solving $f(x) > 0$, the solution set is: $\boxed{(-\infty, -4) \cup (5, \infty)}$

Graph on a number line

Note: If we solve $f(x) \geq 0$, the solution set will be: $(-\infty, -4] \cup [5, \infty)$

E.g. Solve and graph the solution set of:

$$\boxed{x^3 + 3x^2 \leq x + 3}$$

Solution:

Step 1: $\underbrace{\boxed{x^3 + 3x^2 - x - 3}}_{f(x)} \leq 0$ (Get 0 on one side)

Step 2: Set $x^3 + 3x^2 - x - 3 = 0$ (Set $f(x) = 0$)

$\longrightarrow$ Factor by grouping: