$$x^2(x+3)-(x+3)=0$$

$$(x+3)(x^2-1)=0$$

$x+3=0$; $x^2-1=0$

$x=-3$; $x^2=1 \rightarrow x=\pm 1$

Special values: $-3, -1, 1$.

Step3:

Intervals to consider: $(-\infty,-3)$; $(-3,-1)$; $(-1,1)$

and $(1,\infty)$

Step4:

Interval	$(-\infty,-3)$	$(-3,-1)$	$(-1,1)$	$(1,\infty)$
Test Value	-4	-2	0	2
Plug it into $f(x)=x^3+3x^2-x-3$	-15	3	-3	15
Conclusion	$f(x)<0$	$f(x)>0$	$f(x)<0$	$f(x)>0$

Step 5: We are solving $f(x) \leq 0$.

The solution set is $\boxed{(-\infty, -3] \cup [-1, 1]}$

Obj 2: Solve Rational Inequalities.

E.g. Solve and graph the solution set of the inequality:

$$\boxed{\frac{x+1}{x+3} \geq 2}$$

Solution:

Step 1: Express the inequality so that one side is 0 and the other side is a single "fraction":

$$\underbrace{\frac{x+1}{x+3} - 2}_{\text{Need to combine into a single fraction}} \geq 0 \quad \text{(Subtract 2 from both sides)}$$

→ Need LCD. LCD $= (x+3)\cdot 1 = x+3$

$$\frac{x+1}{x+3} - \frac{2\cdot(x+3)}{1\cdot(x+3)} \geqslant 0$$

$$\frac{x+1}{x+3} - \frac{2x+6}{x+3} \geqslant 0$$

$$\frac{x+1-(2x+6)}{x+3} \geqslant 0$$

$$\frac{x+1-2x-6}{x+3} \geqslant 0$$

$$\underbrace{\boxed{\frac{-x-5}{x+3}}}_{f(x)} \geqslant 0$$

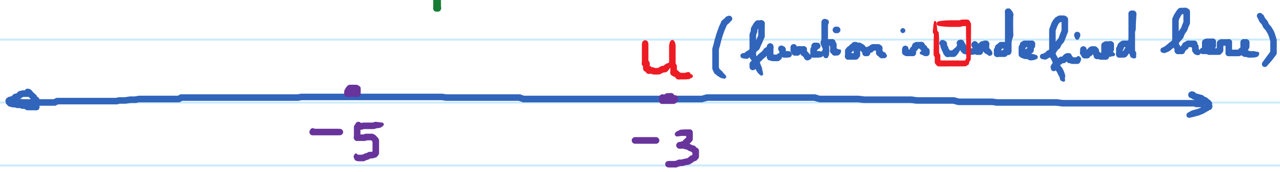
Step 2: Find the "special" values of x by setting both numerator and denominator of $f(x)$ equal to 0 and solve.

$$-x-5=0 \quad ; \quad x+3=0$$

$$\underbrace{\boxed{x=-5} \quad ; \quad \boxed{x=-3}}_{\text{"Special" values of } x.}$$

Thursday, November 14, 2019 10:44 AM

Step 3: Locate these special values on a number line and separate the line into intervals



Intervals to consider: $(-\infty,-5)$; $(-5,-3)$; $(-3,\infty)$

Step 4: Test Values.

Interval	$(-\infty,-5)$	$(-5,-3)$	$(-3,\infty)$
Test Value	-6	-4	0
Plug it into $f(x)=\dfrac{-x-5}{x+3}$	$\dfrac{-(-6)-5}{(-6)+3} = \dfrac{1}{-3} = -\dfrac{1}{3}$	$\dfrac{-(-4)-5}{-4+3} = \dfrac{-1}{-1} = 1$	$\dfrac{-5}{3}$
Conclusion	$f(x)<0$	$f(x)>0$	$f(x)<0$

Step 5: Select the correct intervals.

Since we are solving $f(x)\geq 0$,

Solution set is $\boxed{[-5,-3)}$

we cannot include -3 b/c f is undefined at $x=-3$