Test 3 Review

Wednesday, November 6, 2019 12:37 PM

MC Section.

1 Vertex Formula:

$$\boxed{x_{\text{vertex}} = -\frac{b}{2a} \; ; \; y_{\text{vertex}} = f\left(-\frac{b}{2a}\right)}$$

$$f(x) = 4x^2 + 40x + 96$$

$$a = 4 \; ; \; b = 40 \; ; \; c = 96$$

$$x_{\text{vertex}} = -\frac{40}{2(4)} = -\frac{40}{8} = -5$$

$$y_{\text{vertex}} = 4(-5)^2 + 40(-5) + 96 = -4$$

Vertex $= (-5, -4)$. (A)

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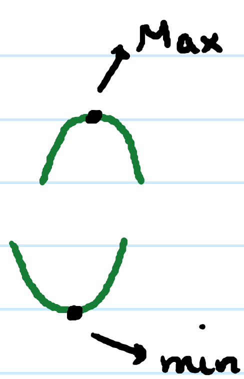
2 $f(x) = ax^2 + bx + c$



f has a MAXIMUM if $a < 0$

f has a MINIMUM if $a > 0$

Maximum (or minimum) value $= y_{\text{vertex}}$

$$f(x) = \frac{1}{2}x^2 - 8x - \frac{11}{2}$$

$a = \frac{1}{2} > 0$. So f has a minimum value.

$$x_{\text{vertex}} = -\frac{b}{2a} = -\frac{(-8)}{2\left(\frac{1}{2}\right)} = 8$$

$$y_{\text{vertex}} = \frac{1}{2}(8)^2 - 8(8) - \frac{11}{2}$$

$$= \frac{1}{2}\cdot 64 - 64 - \frac{11}{2}$$

$$= -32 - \frac{11}{2} = -\frac{75}{2}$$

So, minimum value $= -\frac{75}{2}$ (B)

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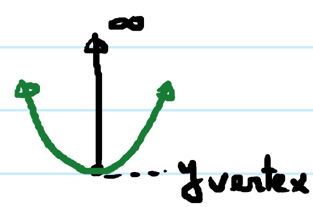
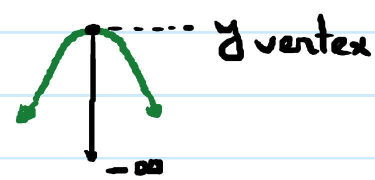
(3)

$$f(x) = ax^2 + bx + c$$

If $a > 0$, then Range $= [y_{\text{vertex}}, \infty)$.



If $a < 0$, then Range $= (-\infty, y_{\text{vertex}}]$



$$f(x) = -4x^2 + 32x - 69$$

Since $a = -4 < 0$, Range $= (-\infty, y \text{ vertex})$

$$x \text{ vertex} = -\frac{b}{2a} = -\frac{32}{2(-4)} = 4$$

$$y \text{ vertex} = -4(4)^2 + 32(4) - 69 = -5$$

So, Range $= (-\infty, -5]$. (B)

4

Leading term test: Leading term $\boxed{a}x^{\boxed{n}} \to$ degree

Leading coeff.

	n even	n odd
$a > 0$	↖ ↗	↙ ↗
$a < 0$	↙ ↘	↖ ↘

$$f(x) = -3x^3 + 2x^2 - 3x - 2$$

Leading term $= -3x^3$ { Degree $= 3$: odd; Leading coeff $= -3 < 0$ }

$\to$ Rises Left, Falls Right. (B)

multiplicity

5) $f(x) = \left(x+\frac{1}{2}\right)^{\boxed{2}}(x+8)^{\boxed{3}}$

To find zeros, set each factor $= 0$ and solve:

$\left(x+\frac{1}{2}\right)^2 = 0$; $(x+8)^3 = 0$

$x + \frac{1}{2} = 0$; $x + 8 = 0$

$\boxed{x = -\frac{1}{2}}$; $\boxed{x = -8}$

Multiplicity $= 2$ (even) Multiplicity $= 3$ (odd)

touches x-axis and bounces back crosses x-axis

(A).

6) Leading term $= \boxed{-0.6}\,x^{\boxed{6}} \to$ Degree : Even

Leading coefficient < 0

$\to$ End Behavior: Falls left, falls right.

(D).

(7) Synthetic Division

$$\begin{array}{r|rrrrrr} -5 & 1 & 8 & 13 & -8 & 8 & -8 \\ & & -5 & -15 & 10 & -10 & 10 \\ \hline & 1 & 3 & -2 & 2 & -2 & \boxed{2} \end{array} \rightarrow \text{Remainder}$$

Quotient $= 1x^4 + 3x^3 - 2x^2 + 2x - 2$

So, Remainder $= 2$.

Quotient $= x^4 + 3x^3 - 2x^2 + 2x - 2$.

(D)

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(8) Since the degree is $n=3$, f has 3 zeros.

2 of the zeros: -6 and i are given.

One zero is hidden.

Since imaginary zeros always occur in conjugate pairs and i is a zero, $-i$ must also be a zero.

All the zeros of f : $-6, i, -i$

Write f in factored form as:

$$f(x) = \boxed{a}(x+6)(x-i)(x+i)$$

leading coefficient (to be determined)

$$= a(x+6)(x^2 - \boxed{i^2})$$

-1

$$= a(x+6)(x^2-(-1))$$

$$= a(x+6)(x^2+1)$$

$$f(x) = a(x^3+6x^2+x+6)$$

Given: $f(-3) = 60$

$$a \cdot \left(\underbrace{(-3)^3 + 6(-3)^2 + (-3) + 6}\right) = 60$$

$$a \cdot (-27 + 54 - 3 + 6) = 60$$

$$a \cdot 30 = 60 \longrightarrow a = \frac{60}{30} = 2$$

$$f(x) = 2(x^3+6x^2+x+6) = \boxed{2x^3+12x^2+2x+12}$$

(c).