

3.3. Dividing Polynomials; Remainder and Factor Theorem

Monday, October 28, 2019 2:35 PM

Obj 1: Divide Polynomials Using Synthetic Division.

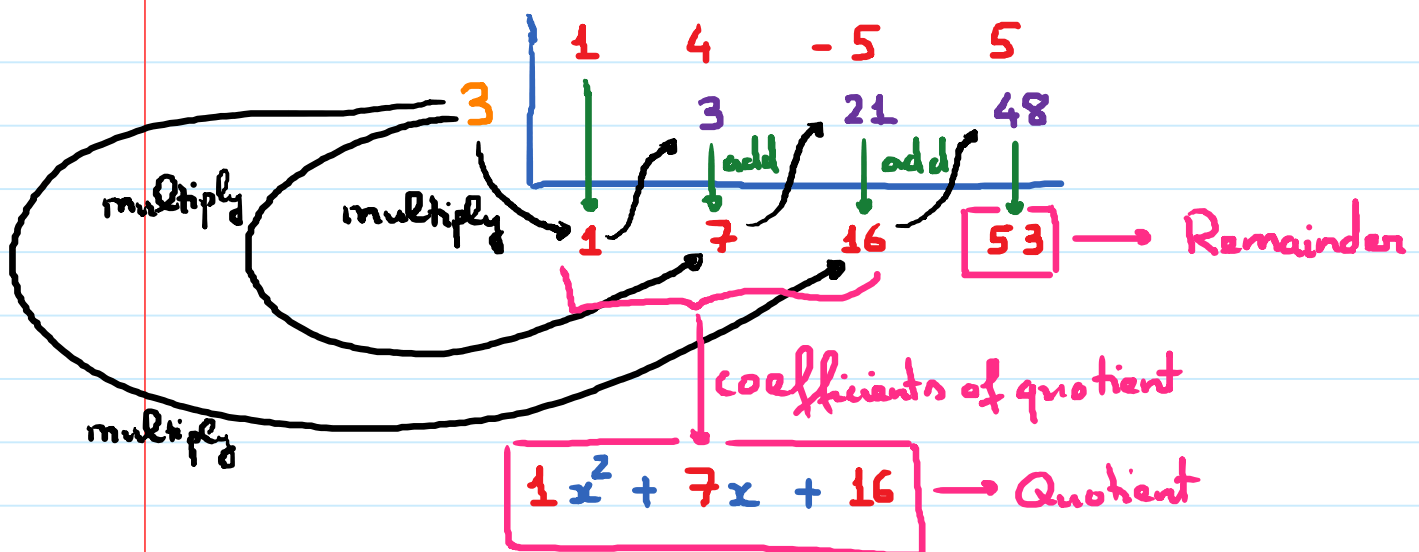
Note: Synthetic division is used to divide a polynomial by $x - c$ or $x + c$

E.g. Use synthetic division to divide

$$\boxed{x^3 + 4x^2 - 5x + 5} \text{ by } \boxed{x - 3}$$

Dividend

Divisor



Result of the division: Quotient = $x^2 + 7x + 16$
Remainder = 53.

How to write the result:

1st way:

$$\boxed{(x^2 + 7x + 16)} \cdot \boxed{(x - 3)} + \boxed{53} = \boxed{x^3 + 4x^2 - 5x + 5}$$

Quotient • Divisor + Remainder = Dividend

2nd way: Dividend

$$\frac{\boxed{x^3 + 4x^2 - 5x + 5}}{\boxed{x - 3}} = \boxed{x^2 + 7x + 16} + \frac{\boxed{53}}{\boxed{x - 3}}$$

Divisor

Quotient

Remainder

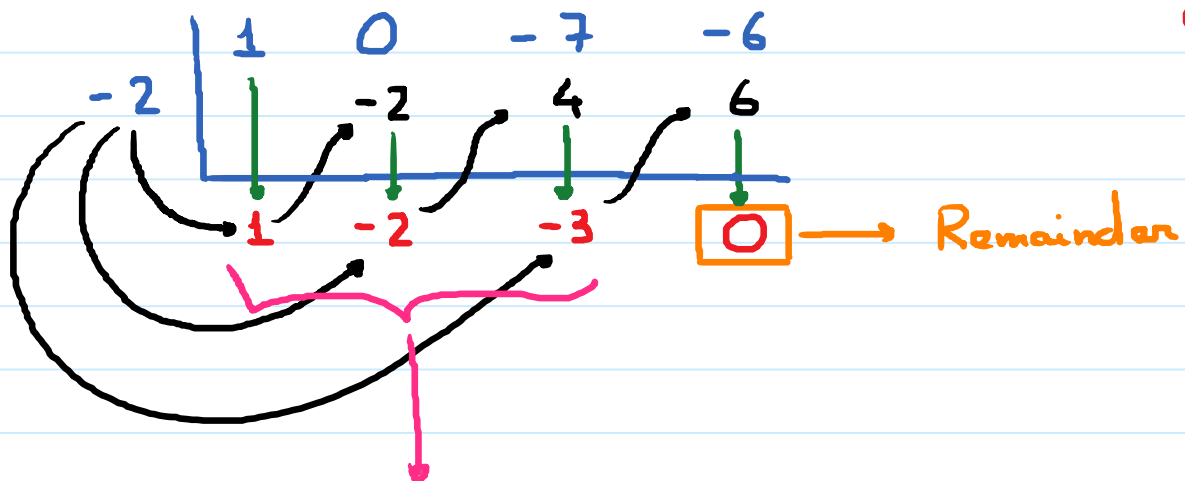
Divisor



E.g. Use synthetic division to divide

$$x^3 - 7x - 6 \text{ by } x + 2$$

Note: x^2 term is missing, put 0 as coeff.



$$\text{Quotient} = 1x^2 - 2x - 3$$

Write result:

$$(x^2 - 2x - 3) \cdot (x + 2) + 0 = x^3 - 7x - 6$$

Quotient \cdot Divisor + Remainder = Dividend

$$\rightarrow (x^2 - 2x - 3)(x + 2) = x^3 - 7x - 6$$

Obj 2: The Remainder Theorem:

If the polynomial $f(x)$ is divided by $x - c$, then the Remainder is equal to $f(c)$.

If the polynomial $f(x)$ is divided by $x + c$, then the Remainder is equal to $f(-c)$.

E.g. When we divide $x^3 + 4x^2 - 5x + 5$ by $x - 3$, we obtained the Remainder = 53.

The Remainder theorem asserts that if we plug 3 into $\underbrace{x^3 + 4x^2 - 5x + 5}_{f(x)}$, that is, if we calculate

$f(3)$, we will get 53.

$$\begin{aligned}\text{Let's check: } f(3) &= (3)^3 + 4 \cdot (3)^2 - 5(3) + 5 \\ &= 27 + 36 - 15 + 5 \\ &= 53.\end{aligned}$$

Obj 3: The Factor Theorem.

Let $f(x)$ be a polynomial.

① If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

② If $x - c$ is a factor of $f(x)$, then $f(c) = 0$

E.g. Consider the equation:

$$\boxed{2x^3 - 3x^2 - 11x + 6 = 0}$$

$f(x)$

Given that $x = 3$ is a solution. Find the remaining solutions.

Sol: $x = 3$ is a solution means that $f(3) = 0$

The factor theorem says that $x-3$ must be a factor of $f(x)$; i.e., $f(x) = (x-3) \cdot (\text{Something})$. Thus, to find the remaining solutions, we just need to divide $f(x)$ by $(x-3)$ (using synthetic division). The quotient will be the remaining factor. Set quotient $= 0$ and solve will give us the remaining solutions.

* $2x^3 - 3x^2 - 11x + 6$ divide by $x - 3$.

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

Quotient: $2x^2 + 3x - 2$.

To find remaining zeros:

$$\text{Set } 2x^2 + 3x - 2 = 0$$

$$(2x - 1)(x + 2) = 0 \quad (\text{Factor})$$

$$2x - 1 = 0 \quad ; \quad x + 2 = 0$$

$$x = \frac{1}{2} \quad ; \quad x = -2.$$

Solution set: $\left\{ 3; \frac{1}{2}, -2 \right\}.$

E.g. Given that $x = -1$ is a solution to:

$$15x^3 + 14x^2 - 3x - 2 = 0.$$

Find the remaining solutions.

Step 1: Use synthetic division to find the remaining factor:

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & 0 \end{array}$$

Quotient = $15x^2 - x - 2$ ← Remaining Factor

Step 2: Set remaining factor = 0

$$15x^2 - x - 2 = 0$$

$$(5x - 2)(3x + 1) = 0$$

$$5x - 2 = 0 \quad ; \quad 3x + 1 = 0$$

$$5x = 2$$

$$3x = -1$$

$$x = \frac{2}{5}$$

$$x = -\frac{1}{3}$$

Solution set:

$$\left\{ -1, \frac{2}{5}, -\frac{1}{3} \right\}$$