3.3. Dividing Polynomials; Remainder and Factor Theorem

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Obj 1: Divide Polynomials Using Synthetic Division.

Note: Synthetic division is used to divide a polynomial by $x - c$ or $x + c$

E.g. Use synthetic division to divide

$$\underbrace{\boxed{x^3 + 4x^2 - 5x + 5}}_{\text{Dividend}} \text{ by } \underbrace{\boxed{x - 3}}_{\text{Divisor}}$$

$$\begin{array}{c|cccc} 3 & 1 & 4 & -5 & 5 \\ & & 3 & 21 & 48 \\ \hline & 1 & 7 & 16 & \boxed{53} \end{array}$$

multiply multiply multiply

add add

$\boxed{53} \longrightarrow$ Remainder

$1, 7, 16$: coefficients of quotient

$\boxed{1x^2 + 7x + 16} \longrightarrow$ Quotient

Result of the division: Quotient $= x^2 + 7x + 16$

Remainder $= 53$.

How to write the result:

1st way:

$$\boxed{(x^2 + 7x + 16)} \cdot \boxed{(x - 3)} + \boxed{53} = \boxed{x^3 + 4x^2 - 5x + 5}$$

Quotient · Divisor + Remainder = Dividend

2nd way:

$$\frac{\overbrace{\boxed{x^3 + 4x^2 - 5x + 5}}^{\text{Dividend}}}{\underbrace{\boxed{x - 3}}_{\text{Divisor}}} = \underbrace{\boxed{x^2 + 7x + 16}}_{\text{Quotient}} + \frac{\overbrace{\boxed{53}}^{\text{Remainder}}}{\underbrace{\boxed{x - 3}}_{\text{Divisor}}}$$

E.g. Use synthetic division to divide

$x^3 - 7x - 6$ by $x + 2$

Note: x^2 term is missing, put 0 as coeff.

$$
\begin{array}{r|rrrr}
 & 1 & 0 & -7 & -6 \\
-2 & & -2 & 4 & 6 \\
\hline
 & 1 & -2 & -3 & \boxed{0}
\end{array}
\longrightarrow \text{Remainder}
$$

Quotient = $1x^2 - 2x - 3$

Write result:

$$\boxed{(x^2 - 2x - 3)} \cdot \boxed{(x+2)} + \boxed{0} = \boxed{x^3 - 7x - 6}$$

Quotient · Divisor + Remainder = Dividend

$\rightarrow (x^2 - 2x - 3)(x + 2) = x^3 - 7x - 6$

Obj 2: The Remainder Theorem:

If the polynomial $f(x)$ is divided by $x - c$, then the Remainder is equal to $f(c)$.

If the polynomial $f(x)$ is divided by $x + c$, then the Remainder is equal to $f(-c)$.

E.g. When we divide $x^3 + 4x^2 - 5x + 5$ by $x - 3$, we obtained the Remainder = 53.

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The Remainder theorem asserts that if we plug 3 into $\underbrace{x^3 + 4x^2 - 5x + 5}_{f(x)}$, that is, if we calculate $f(3)$, we will get 53.

Let's check: $f(3) = (3)^3 + 4\cdot(3)^2 - 5(3) + 5$

$= 27 + 36 - 15 + 5$

$= 53.$

Obj 3: The Factor Theorem.

Let $f(x)$ be a polynomial.

(a) If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

(b) If $x - c$ is a factor of $f(x)$, then $f(c) = 0$

E.g. Consider the equation:

$$\boxed{2x^3 - 3x^2 - 11x + 6} = 0$$

$f(x)$

Given that $x = 3$ is a solution. Find the remaining solutions.

Sol: $x = 3$ is a solution means that $f(3) = 0$

The factor theorem says that $x-3$ must be a factor of $f(x)$; i.e., $f(x) = (x-3)\cdot(\text{Something})$

Thus, to find the remaining solutions, we just need to divide $f(x)$ by $(x-3)$ (using synthetic division). The quotient will be the remaining factor. Set quotient $=0$ and solve will give us the remaining solutions.

* $2x^3 - 3x^2 - 11x + 6$ divide by $x-3$.

$$\begin{array}{c|cccc} & 2 & -3 & -11 & 6 \\ 3 & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

Quotient: $2x^2 + 3x - 2$.

To find remaining zeros:

Set $2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$ (Factor)

$2x - 1 = 0$; $x + 2 = 0$

$x = \frac{1}{2}$; $x = -2$.

Solution set: $\boxed{\{3; \frac{1}{2}, -2\}}$.

E.g. Given that $x = -1$ is a solution to:

$$15x^3 + 14x^2 - 3x - 2 = 0.$$

Find the remaining solutions.

Step 1: Use synthetic division to find the remaining factor:

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & 0 \end{array}$$

Quotient $= \boxed{15x^2 - x - 2}$ ⟵ Remaining Factor

Step 2: Set remaining factor $= 0$

$$15x^2 - x - 2 = 0$$

$$(5x - 2)(3x + 1) = 0$$

$5x - 2 = 0$; $3x + 1 = 0$

$5x = 2$ $\quad$ $3x = -1$

$x = \frac{2}{5}$ $\quad$ $x = -\frac{1}{3}$

Solution set: $\boxed{\left\{-1, \frac{2}{5}, -\frac{1}{3}\right\}}$