

Monday, November 18, 2019 12:47 PM

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Obj 1: Exponential Functions and their graphs.

The exponential function with base b is the function of the form

$$f(x) = b^x \quad \text{or} \quad y = b^x$$

Here: b is a constant, b is positive ($b > 0$);

$b \neq 1$. b is called the base of the function

E.g. $f(x) = 2^x$; $f(x) = \left(\frac{1}{2}\right)^x$

$\downarrow$ $\downarrow$

base = 2 base = $\frac{1}{2}$

$$f(x) = (2.71819)^x$$

↓
base

$f(x) = \boxed{3}^{x+1}$; $f(x) = 7.5 (\boxed{3})^{x-1}$

$\downarrow$ $\downarrow$

base base = 3

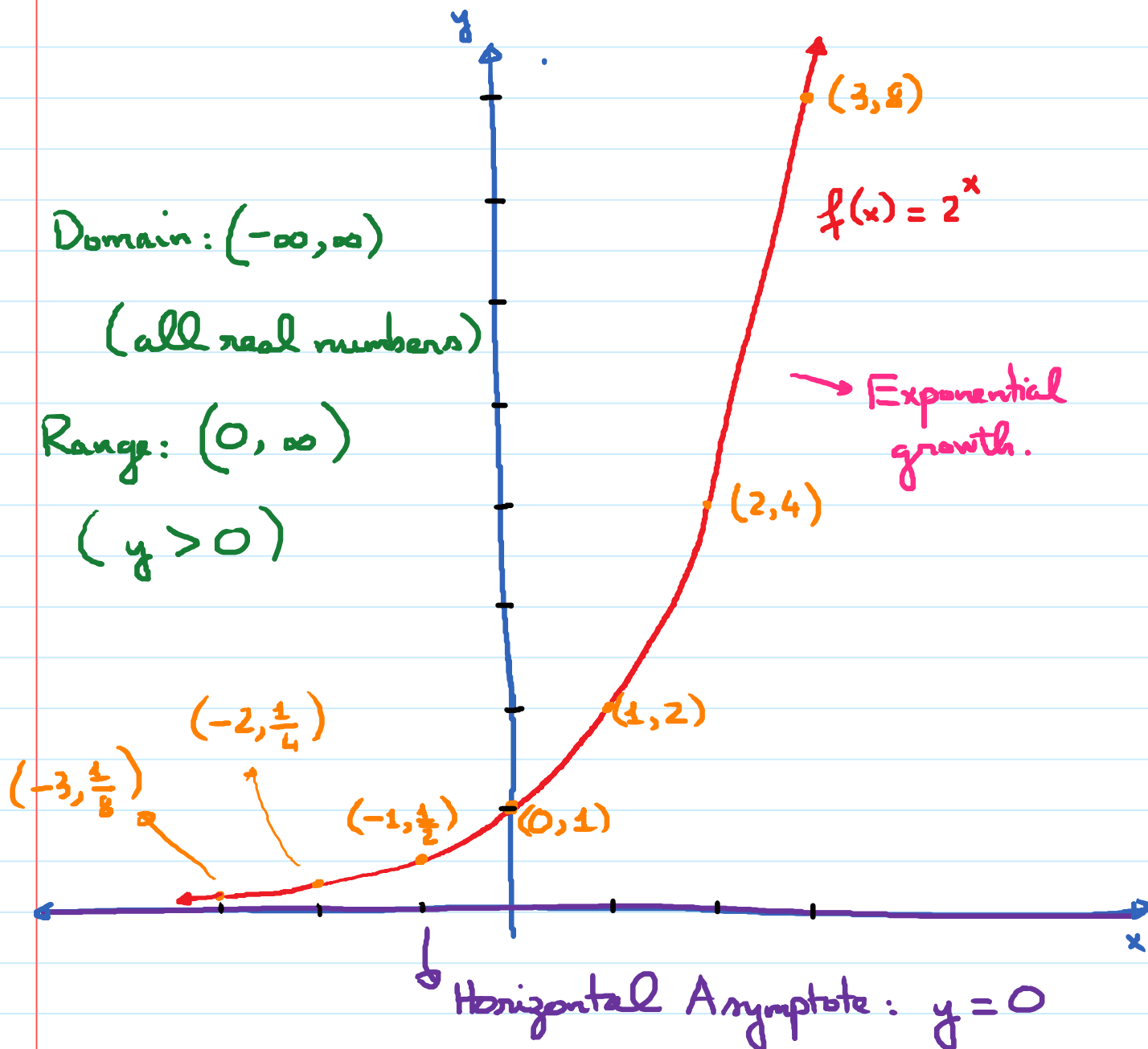
These are exponential functions.

Note: In calculator, use  to do exponents.

* Graphs of Exponential Functions.

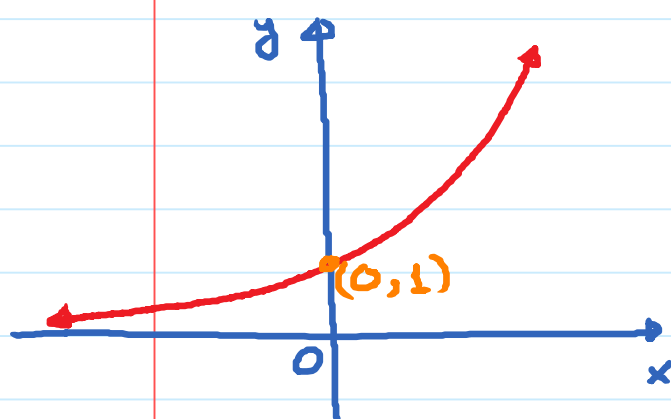
E.g. $f(x) = 2^x$

x	$f(x) = 2^x$	Point
-3	$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$(-3, \frac{1}{8})$
-2	$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$(-2, \frac{1}{4})$
-1	$f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$	$(-1, \frac{1}{2})$
0	$f(0) = 2^0 = 1$	$(0, 1)$
1	$f(1) = 2^1 = 2$	$(1, 2)$
2	$f(2) = 2^2 = 4$	$(2, 4)$
3	$f(3) = 2^3 = 8$	$(3, 8)$



In general, the graph of $f(x) = b^x$ has the following properties:

Base $b > 1$



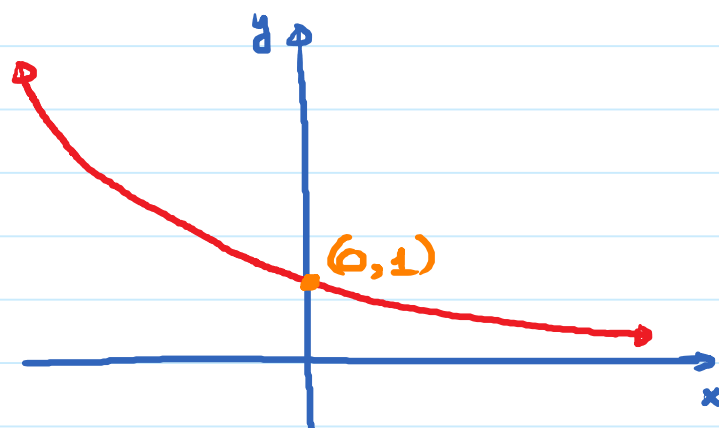
Horizontal asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Exponential growth

Base: $0 < b < 1$



$y = 0$

$(-\infty, \infty)$

$(0, \infty)$

Exponential Decay.

Obj 2: Transformations of Exponential Functions.

Transformation	Equation	Description.
Vertical translation	$y = b^x + c$	up, c units
	$y = b^x - c$	down, c units
Horizontal translation	$y = b^{x+c}$	left, c units
	$y = b^{x-c}$	Right, c units.
Vertical Stretch or shrink	$y = c \cdot b^x$	$c > 1$: stretch
		$0 < c < 1$: shrink
Reflection	$y = -b^x$	Flip across x -axis

E.g. Find domain, range, asymptote, y -intercept of the given function.

(a) $f(x) = 2^x - 7$; (b) $g(x) = -3^{x+1}$

Sol:

① $f(x) = 2^x - 7$ (Parent function $y = 2^x$)

Domain: $(-\infty, \infty)$

Range: $(-7, \infty)$ (Shift down 7 units)

H.A.: $y = -7$ (parent: $y = 0$)

y-intercept: $(0, -6)$ (parent: $(0, 1)$)

② $g(x) = -3^{x+1}$ (Parent function $y = 3^x$)

Domain: $(-\infty, \infty)$ (Shift left 1, flip

Range: $(-\infty, 0)$ across x-axis)

H.A.: $y = 0$

y-intercept: Set $x=0$ in equation.

$$y = -3^{0+1} = -3^1 = -3$$

$$(0, -3)$$

Obj 3: The natural base e .

In many applications of exponential function, the base of the function equal to a special decimal number, called e .

$$e \approx 2.718281827 \dots$$

Graph of $f(x) = e^x$ is exp. growth b/c $e > 1$.