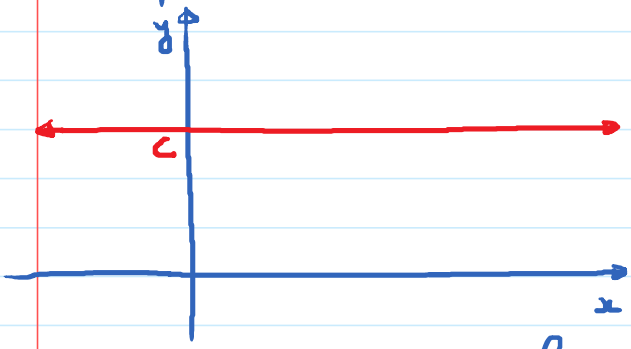
Section 2.5. Transformations of Functions

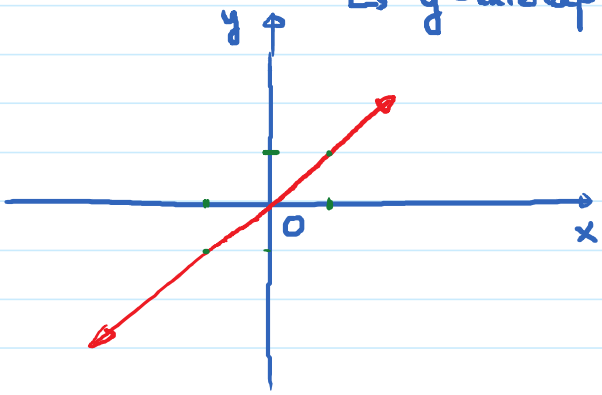
Wednesday, September 25, 2019 12:35 PM

Objective 1: Graphs of Common Functions.

(1) $f(x) = c$; c is a constant.

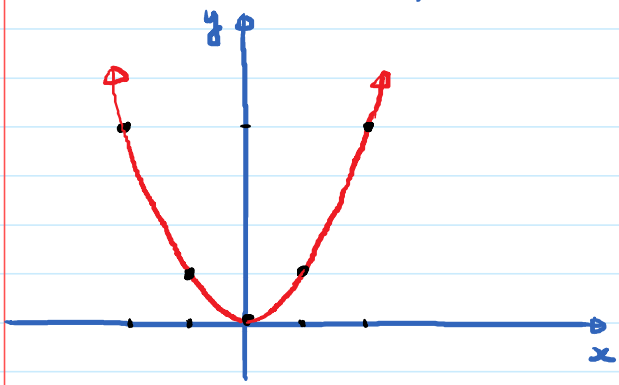


(2) $f(x) = x$ → slope = 1; → y-intercept: (0, 0)



(3) $f(x) = x^2$. Graph is a parabola.

x	$y = f(x) = x^2$	Point (x, y)
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$

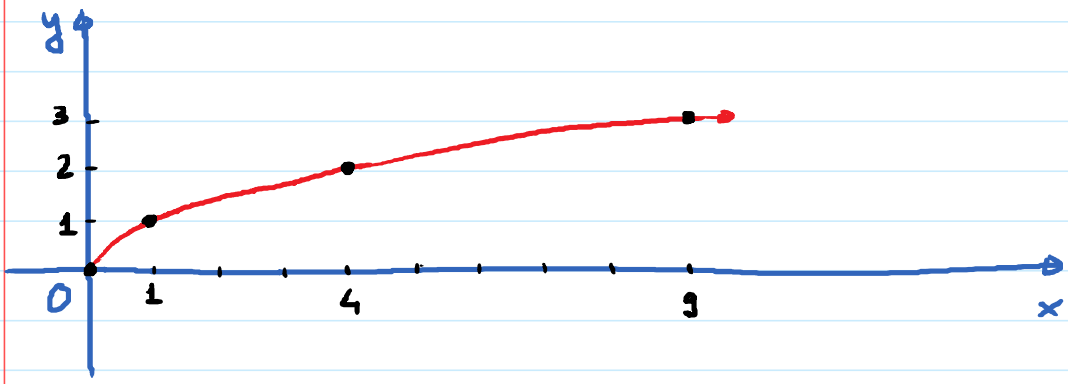


Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(4) $f(x) = \sqrt{x}$

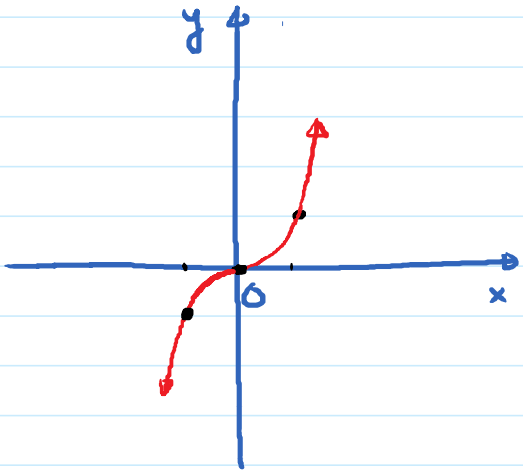
x	$y = f(x) = \sqrt{x}$	Point (x, y)
0	0	(0, 0)
1	1	(1, 1)
4	2	(4, 2)
9	3	(9, 3)



Domain: $[0, \infty)$. Range: $[0, \infty)$

(5) $f(x) = x^3$

x	$y = f(x) = x^3$	Point (x, y)
−1	−1	(−1, −1)
0	0	(0, 0)
1	1	(1, 1)



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

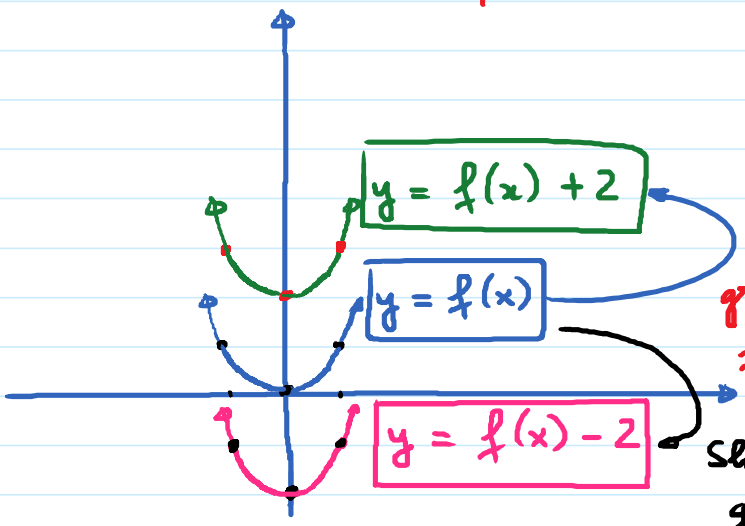
Objective 2: Vertical Shifts.

E.g. $y = f(x) = x^2$ ← parent function

Graph $y = f(x) + 2 = x^2 + 2$

x	$y = f(x) = x^2$
-1	1
0	0
1	1

x	$y = f(x) + 2 = x^2 + 2$
-1	3
0	2
1	3



x	$y = f(x) - 2 = x^2 - 2$
-1	-1
0	-2
1	-1

Vertical Shifts:

Given the function $y = f(x)$ and a positive constant c.

① The graph of $\boxed{y = f(x) + c}$ is the graph of the function $y = f(x)$ shifted up by c units.

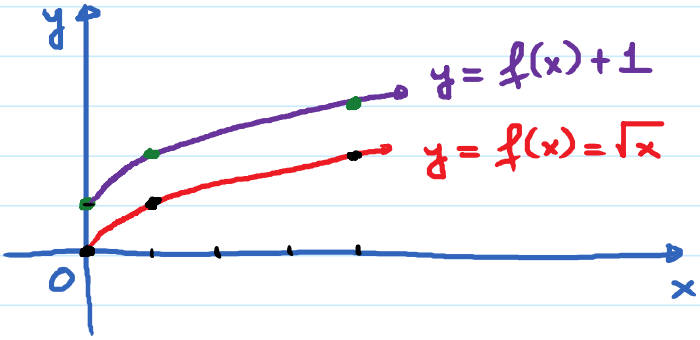
(2) The graph of $\boxed{y = f(x) - c}$ is the graph of $y = f(x)$ shifted down by c units.

Ex: Graph the function $y = f(x) = \sqrt{x}$. (3 Key Points). Then use transformation to graph

$y = \sqrt{x} + 1$.

Solution:

x	$y = f(x)$
0	0
1	1
4	2



Objective 3: Horizontal Shifts.

E.g. Given $y = f(x) = x^2$.

(1) What is the formula for $y = f(x-3)$?

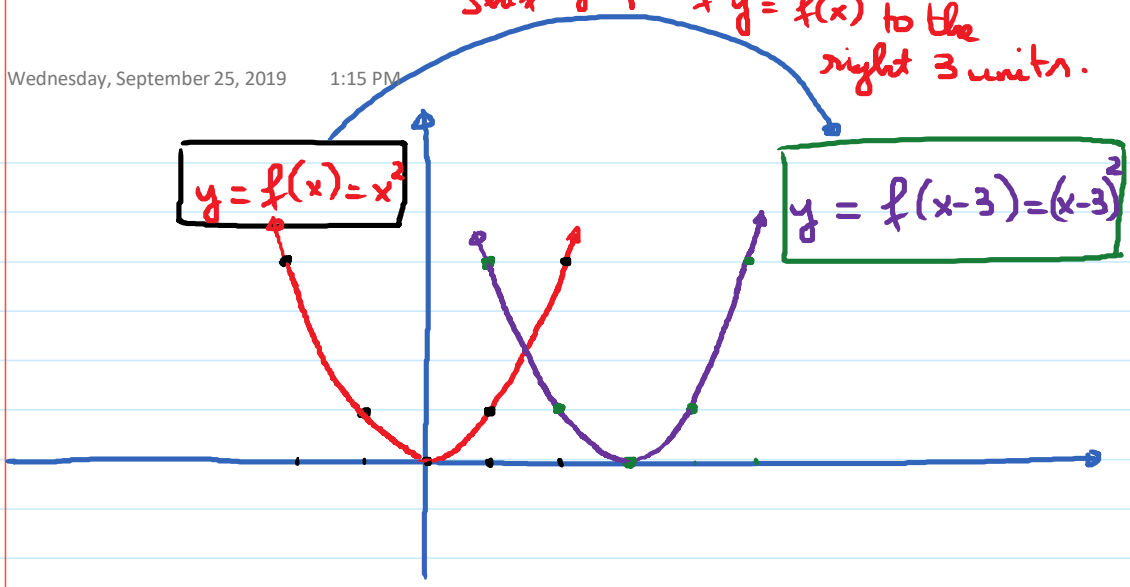
$y = (x-3)^2$ (Replace x by $x-3$ in the original equation)

(2) Graph $y = f(x-3) = (x-3)^2$

x	$y = f(x) = x^2$	(x, y)
-2	4	(-2, 4)
-1	1	(-1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)

x	$y = f(x-3) = (x-3)^2$	(x, y)
1	$(1-3)^2 = 4$	(1, 4)
2	$(2-3)^2 = 1$	(2, 1)
3	$(3-3)^2 = 0$	(3, 0)
4	$(4-3)^2 = 1$	(4, 1)
5	$(5-3)^2 = 4$	(5, 4)

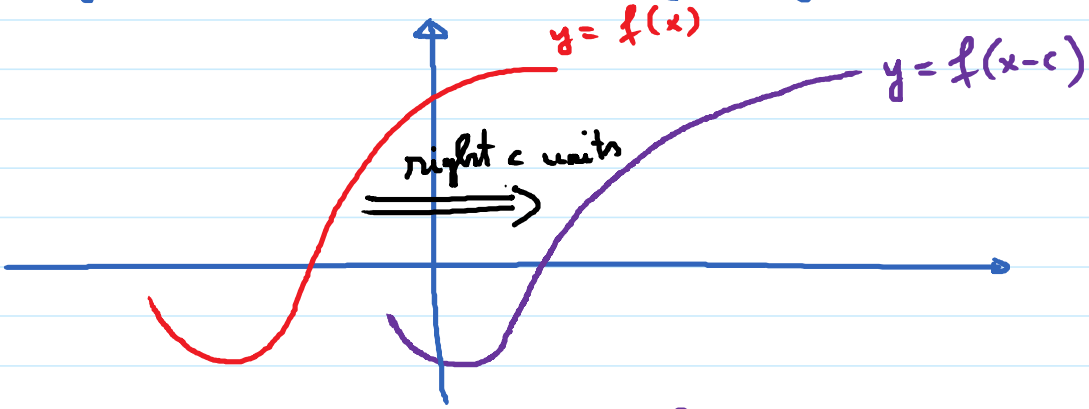
Move right 3 units



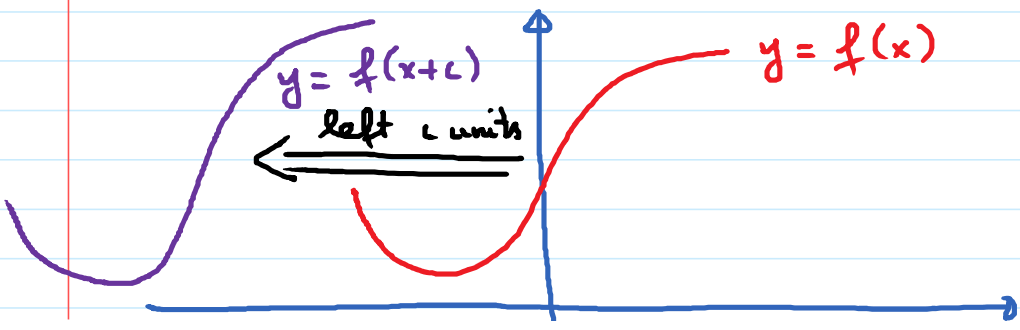
Horizontal Shifts.

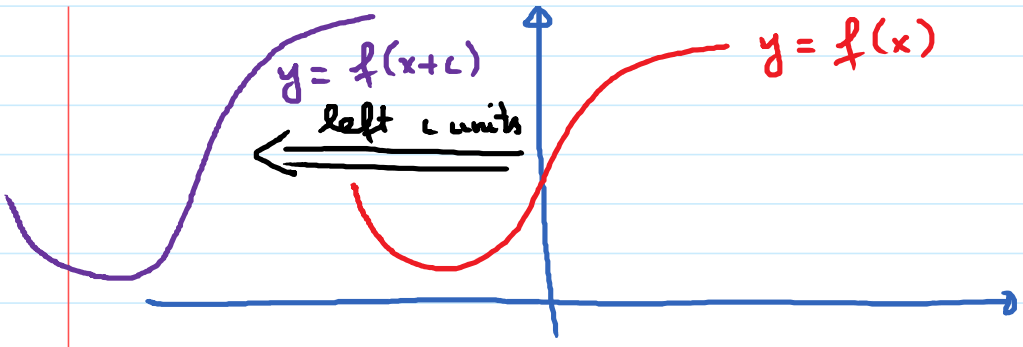
Given a function $y = f(x)$ and a positive number c

(1) The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right by c units.



(2) The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left by c units.




y = f(x+c)
left c units
y = f(x)

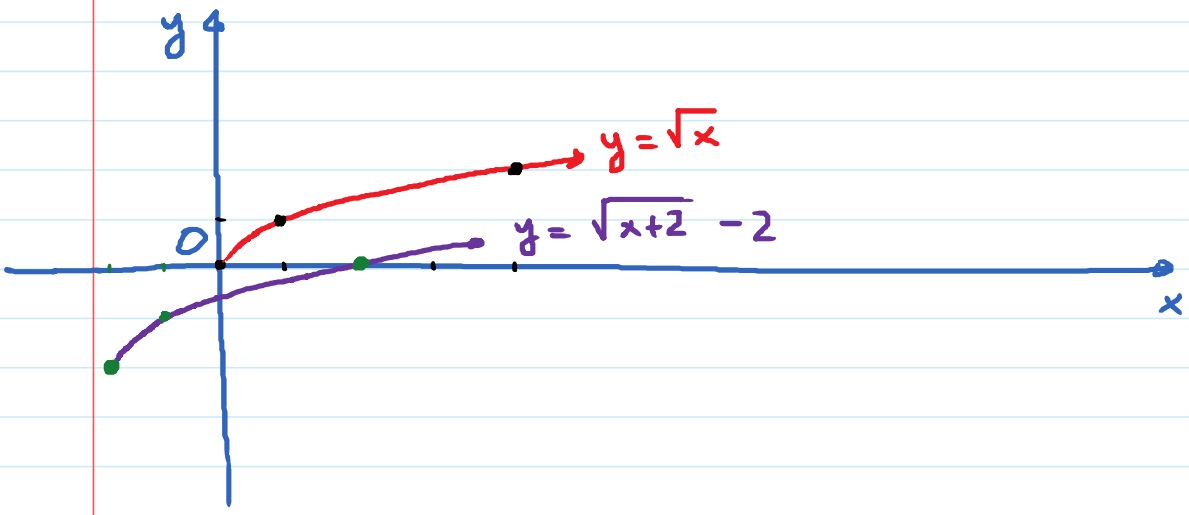
E.g. Combination of vertical shifts and horizontal shifts.

(1) Graph the function $y = \sqrt{x}$ (3 key points).

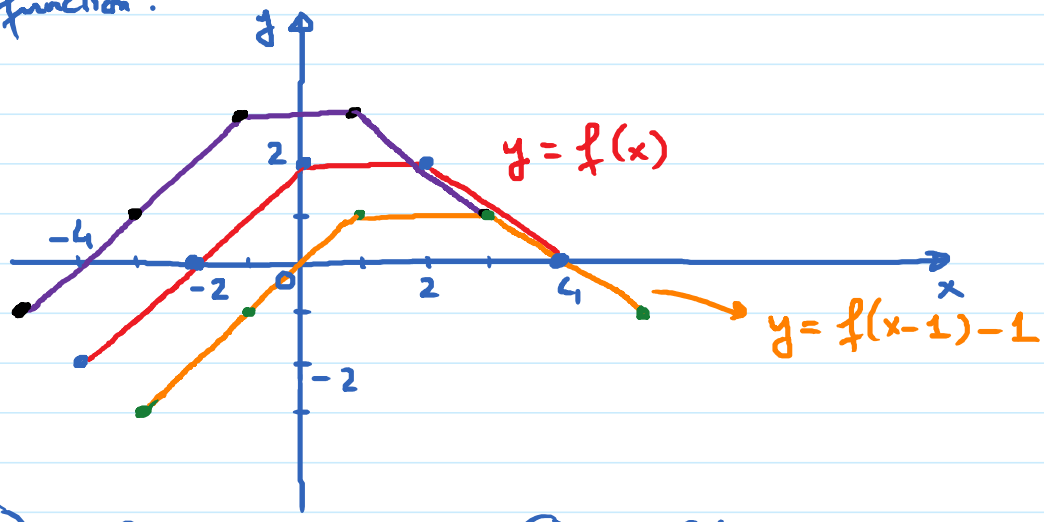
(2) Use transformation to graph $y = \sqrt{x+2} - 2$.

$y = \sqrt{x} \xrightarrow[\text{left } 2]{} \sqrt{x+2} \xrightarrow[\text{down } 2]{} \sqrt{x+2} - 2$

x	$y = \sqrt{x}$	(x, y)		
0	0	$(0,0)$	$\xrightarrow[\text{left } 2]{}$ $(-2,0)$	$\xrightarrow[\text{down } 2]{}$ $(-2,-2)$
1	1	$(1,1)$	$\longrightarrow (-1,1)$	$\longrightarrow (-1,-1)$
4	2	$(4,2)$	$\longrightarrow (2,2)$	$\longrightarrow (2,0)$



E.g. Use the graph of $y = f(x)$ to graph the new function.



① $y = f(x-1) - 1$

1 Right, 1 Down.

② $y = f(x+1) + 1$

1 left 1 up.