Section 2.6: Combinations of Functions; Composite Function

Wednesday, October 2, 2019 12:32 PM

Objective: Find a function's Domain.

Key: The domain of a function $y = f(x)$ is the set of all real numbers: $(-\infty, \infty)$ unless x appears in the denominator or x appears in a square root (or an even root).

E.g. $f(x) = x^2 - 7x + 12$

This function contains neither a denominator nor a square root.

So, the domain of f is the set of all real numbers.

Domain of $f = (-\infty, \infty)$

E.g. Function has x in the denominator.

Key: To find the domain of such a function, we need to find the values of x that cause the denominator equal to zero. The domain will be the set of real numbers excluding those values.

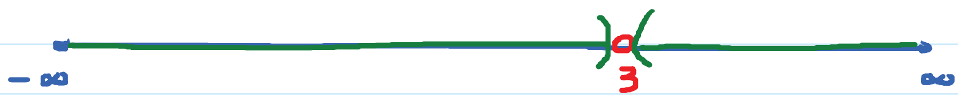
Find the domain:

1. $f(x) = \dfrac{1}{x-3}$.

Step 1: Set denominator $= 0$ and solve for x.

$$\underbrace{x-3}_{\text{denom.}} = 0 \longrightarrow x = 3.$$

Step 2: Domain is the set of real numbers excluding the values from step 1. Write the answer in interval notation or set notation.



exclude from domain

Interval Notation: $\boxed{(-\infty, 3) \cup (3, \infty)}$

union

Set notation: $\boxed{\{x \mid x \neq 3\}}$

such that

(2) $g(x) = \dfrac{3x+2}{\boxed{x^2 - 2x - 3}}$ $\longrightarrow$ denominator

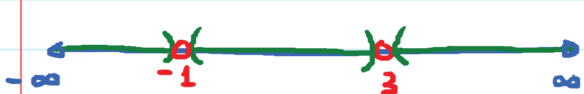
Step 1: Set Denominator $= 0$ and solve for x.

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0 \quad \text{(Factor)}$$

$x - 3 = 0$	$x + 1 = 0$
$x = 3$	$x = -1$

Step 2: Write Domain in interval notation or set notation.



$\boxed{D = (-\infty, -1) \cup (-1, 3) \cup (3, \infty)}$

(interval notation)

Set notation: $\{x \mid x \neq -1,\ x \neq 3\}$

(3) $h(x) = \dfrac{5x}{\boxed{x^2 - 49}}$ → Denominator

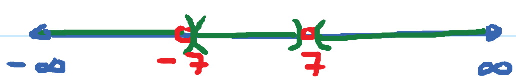
Step 1: Set denom. $= 0$ and solve for x

$$x^2 - 49 = 0$$

$$(x-7)(x+7) = 0$$

$x - 7 = 0$	$x + 7 = 0$
$x = 7$	$x = -7$

Step 2. Answer.



$\boxed{D = (-\infty, -7) \cup (-7, 7) \cup (7, \infty)}$

Set Notation: $\boxed{\{x \mid x \neq 7\ ;\ x \neq -7\}}$

E.g. Functions that have x in a square root

Key: The expression under the square root sign cannot be negative. To find the domain, we take the expression under the square root sign and set it ≥ 0 and solve the resulting inequality. The solution to the inequality is the domain.

Find the domain

① $f(x) = \sqrt{\boxed{3x + 12}} \longrightarrow$ expression under the square root

Step 1: Set expression under square root ≥ 0

$$3x + 12 \geq 0$$

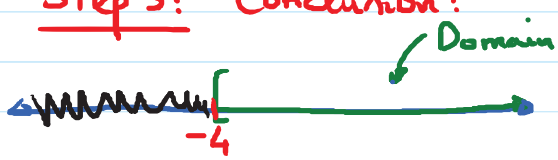
Step 2: Solve the inequality.

$$3x \geq -12 \quad (\text{Subtract } 12)$$

$$x \geq -\frac{12}{3} \quad (\text{Divide by } 3)$$

$$x \geq -4$$

Step 3: Conclusion:



$D = [-4, \infty)$

(interval notation)

Set notation: $D = \{x \mid x \geq -4\}$

② $g(x) = \sqrt{27 - 9x}$

Step 1: Set expression under square root ≥ 0

$$27 - 9x \geq 0$$

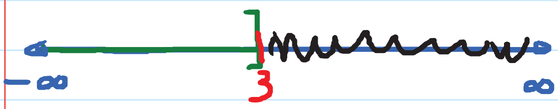
Step 2: Solve the inequality

$$-9x \geq -27 \quad (\text{Subtract})$$

$$\frac{\cancel{-9}x}{\cancel{-9}} \leq \frac{-27}{-9} \quad (\text{Divide by } -9)$$

$x \leq 3$

Step 3: Conclusion.



$\boxed{D = (-\infty, 3]}$

In set notation : $\boxed{D = \{x \mid x \leq 3\}}$

E.g. Functions that have x in a square root that is in the denominator.

Key: Set the expression under the square root > 0 (since the square root is in the denominator, it cannot be equal to 0)

Find the domain: ① $f(x) = \dfrac{3x+2}{\sqrt{2x-14}}$

Step 1: $2x - 14 > 0$

Step 2: $2x > 14 \rightarrow x > \dfrac{14}{2} \rightarrow x > 7$

Step 3: Conclusion: $\boxed{D = (7, \infty)}$

② $j(x) = \dfrac{5x}{\sqrt{24-3x}}$

$\boxed{D = (-\infty, 8)}$

Step 1: $24 - 3x > 0$

Step 2: $-3x > -24 \rightarrow x < \dfrac{-24}{-3} \rightarrow x < 8$

Objective 2: Combine functions (Sum, Difference, product, quotient)

Given 2 functions f and g

1. Sum of f and g:
   - Notation: $f+g$
   - Formula: $(f+g)(x) = f(x) + g(x)$

   (add the formula for f and g and simplify)

2. Difference of f and g:
   - Notation: $f-g$
   - Formula: $(f-g)(x) = f(x) - g(x)$

   (Subtract and Simplify)

3. Product of f and g:
   - Notation: $f \cdot g$
   - Formula: $(f \cdot g)(x) = f(x) \cdot g(x)$

   (Multiply and Simplify)

4. Quotient of f and g:
   - Notation: $\frac{f}{g}$
   - Formula: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

   (Divide and Simplify)