Note:

To find the domain in Case 1, 2 and 3; i.e., domain of $f+g$; $f-g$ or $f\cdot g$:

Step 1: Find the domain of f: D_f

Find the domain of g: D_g

Step 2: The domain of the combined function is the intersection of domain of f and the domain of g: $D_f \cap D_g$

intersection.

Note: To find the domain of $\frac{f}{g}$, we still need to find D_f; D_g and the intersection of D_f and D_g. In addition to that, we need to solve for x such that $g(x) = 0$ and add the further restriction (excluding these values) to $D_f \cap D_g$.

E.g. Given $f(x) = 2x - 1$ and $g(x) = x^2 + x - 2$.

Q 1: Find each of the following functions:

(a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $(fg)(x)$

(d) $\left(\frac{f}{g}\right)(x)$

Q2: Find domain of each function.

Sol:

Q1: (a) $(f+g)(x) = f(x) + g(x)$

$$= \underbrace{(2x-1)}_{f(x)} + \underbrace{(x^2+x-2)}_{g(x)}$$

$$\boxed{(f+g)(x) = x^2+3x-3}$$

(b) $(f-g)(x) = f(x) - g(x)$

$$= \underbrace{(2x-1)}_{f(x)} - \underbrace{(x^2+x-2)}_{g(x)}$$

$$= 2x - 1 - x^2 - x + 2$$

$$= -x^2 + x + 1$$

$$\boxed{(f-g)(x) = -x^2 + x + 1}$$

(c) $(fg)(x) = f(x) \cdot g(x)$

$$= \underbrace{(2x-1)}_{f(x)}\underbrace{(x^2+x-2)}_{g(x)}$$

$$= 2x^3 + 2x^2 - 4x - x^2 - x + 2$$

$$= 2x^3 + x^2 - 5x + 2$$

$$\boxed{(fg)(x) = 2x^3 + x^2 - 5x + 2}$$

$f(x)$

$$\text{(d)}\quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\boxed{2x-1}}{\boxed{x^2+x-2}} = \frac{2x-1}{(x+2)(x-1)}$$

$g(x)$

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{2x-1}{(x+2)(x-1)}}$$

* Find domain of $f+g$; $f-g$; fg

<u>Step 1:</u> Find Domain of f ; D_f and Domain of g ; D_g

$f(x) = 2x-1$. $D_f = (-\infty, \infty)$

$g(x) = x^2+x-2$. $D_g = (-\infty, \infty)$ intersection

<u>Step 2:</u> Find the intersection : $D_f \cap D_g$

D_f : $-\infty$ ⟵⟶ ∞

D_g : $-\infty$ ⟵⟶ ∞

The domain of $f+g$, $f-g$, fg is $(-\infty, \infty)$

* Find domain of $\frac{f}{g}$

- Do the same Step 1 and Step 2 as the above

<u>Step 3:</u> Set $g(x) = 0$ and solve for x.

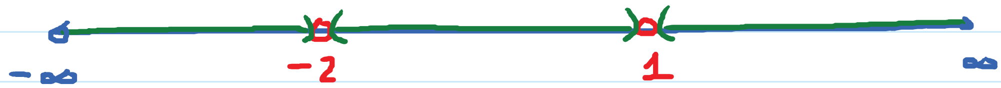
The domain of $\frac{f}{g}$ will be the previous answer excluding the values from step 3.

$g(x) = 0 \rightarrow x^2 + x - 2 = 0$

$\rightarrow (x-1)(x+2) = 0$

$\rightarrow x = 1 \; ; \; x = -2.$

Domain of $\frac{f}{g}$ = $\boxed{(-\infty, -2) \cup (-2, 1) \cup (1, \infty)}$



E.g. Given $f(x) = \sqrt{x-3}$ and $g(x) = \sqrt{x+1}$

(a) Find $(f+g)(x)$

(b) Find $(f+g)(3)$

(c) Find domain of $f+g$.

(a) $(f+g)(x) = f(x) + g(x)$

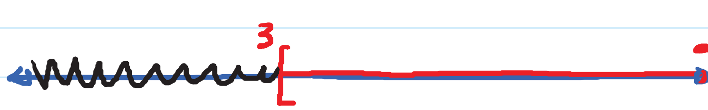
$= \boxed{\sqrt{x-3} + \sqrt{x+1}}$

(b) $(f+g)(3) = \sqrt{3-3} + \sqrt{3+1} = \sqrt{0} + \sqrt{4} = \boxed{2}$

(c) Domain of $f+g$.

Step 1: Find D_f and D_g

$f(x) = \sqrt{x-3}$. Set $x - 3 \geq 0 \rightarrow x \geq 3$

D_f : 

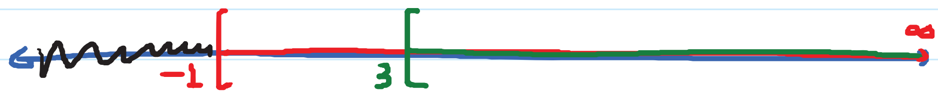
$g(x) = \sqrt{x+1}$. $x + 1 \geq 0 \to x \geq -1$

D_g :

Step 2: Find the intersection $D_f \cap D_g$

D_f

D_g



Answer: $D_f \cap D_g = [3, \infty)$

So, Domain of $f+g = \boxed{[3, \infty)}$

Obj 3: Composite Functions

Composition of Functions (Composite Functions)

Given functions f and g. The composition of f and g is a function. This function is denoted by $f \circ g$ (read as f circle g or f of g) and it is defined by the equation:

$$\boxed{(f \circ g)(x) = f(\underbrace{g(x)}))}$$

plug $g(x)$ into f

E.g. Given $f(x) = 3\boxed{x} - 4$; $g(x) = \boxed{x^2 - 2x + 6}$

Find the following

ⓐ $(f \circ g)(x)$ ⓑ $(g \circ f)(x)$

ⓒ $(f \circ g)(1)$ ⓓ $(g \circ f)(1)$

Sol:

ⓐ $(f \circ g)(x) = f(g(x))$

$= f(\underbrace{x^2 - 2x + 6})$ (Replace $g(x)$ by its formula)

$= 3(x^2 - 2x + 6) - 4$ (Substitute the formula of $g(x)$ into every x in the formula of f)

$= 3x^2 - 6x + 18 - 4$ (Distribute)

$$\boxed{(f \circ g)(x) = 3x^2 - 6x + 14}$$

ⓑ $(g \circ f)(x) = g(f(x))$ (Plug $f(x)$ into g)

$= g(3x - 4)$

$= (3x - 4)^2 - 2(3x - 4) + 6$

$= (3x - 4)(3x - 4) - 6x + 8 + 6$

$$= 9x^2 - 12x - 12x + 16 - 6x + 14$$

$$\boxed{(g \circ f)(x) = 9x^2 - 30x + 30}$$

(c) $(f \circ g)(1) = ?$

We obtained the formula for $(f \circ g)(x)$ in (a)

$(f \circ g)(x) = 3x^2 - 6x + 14.$

Now, we just need to plug $x = 1$ into this formula

$$(f \circ g)(1) = 3(1)^2 - 6(1) + 14 = \boxed{11}$$

2nd way to solve this:

$$(f \circ g)(1) = f(g(1))$$

what you get after you plug 1 into g, you plug that into f

plug 1 into g

Step 1: Plug 1 into g.

$$g(1) = (1)^2 - 2(1) + 6 = \boxed{5}$$

Step 2: Plug the answer in Step 1 into f

$$f(g(1)) = 3(5) - 4 = \boxed{11}$$

(d) $(g \circ f)(1)$

Method 1: Plug $x = 1$ into formula for $(g \circ f)(x)$

$$(g \circ f)(1) = 9(1)^2 - 30(1) + 30$$

$$= 9 - 30 + 30 = \boxed{9}$$

Method 2: Plug 1 into f. Then plug f(1) into g

$f(1) = 3(1) - 4 = -1$

$g(f(1)) = g(-1)$

$= (-1)^2 - 2(-1) + 6$

$= 1 + 2 + 6 = \boxed{9}$