Section 2.7. Inverse Functions

Wednesday, October 9, 2019 12:34 PM

Obj 1: Verify Inverse Functions

Recall from Section 2.6: if f and g are functions then we can find the composition of f and g by plugging $g(x)$ into f.

We can also find the composition of g and f by plugging $f(x)$ into g.

If after we plug in and simplify, we obtain x, then we say that f and g are inverse functions of one another.

Definition of the Inverse of a Function.

If f and g are functions such that:

$$\boxed{f(g(x)) = x}$$

and

$$\boxed{g(f(x)) = x}$$

then we say that the function g is the inverse of the function f. We denote it by f^{-1} (read as f inverse).

E.g. Given $f(x) = 3x + 2$

$$g(x) = \frac{x-2}{3}$$

Q: Verify that the functions are inverse functions of one another.

$$f(\boxed{g(x)}) = f\left(\frac{x-2}{3}\right) = \cancel{3}\left(\frac{x-2}{\cancel{3}}\right) + 2$$

$$= x \cancel{-2} \cancel{+2} = x$$

So, $f(g(x)) = x$

$$g(\boxed{f(x)}) = g(3x+2) = \frac{(3x+2) - 2}{3}$$

$$= \frac{3x \cancel{+2} \cancel{-2}}{3} = \frac{\cancel{3}x}{\cancel{3}} = x$$

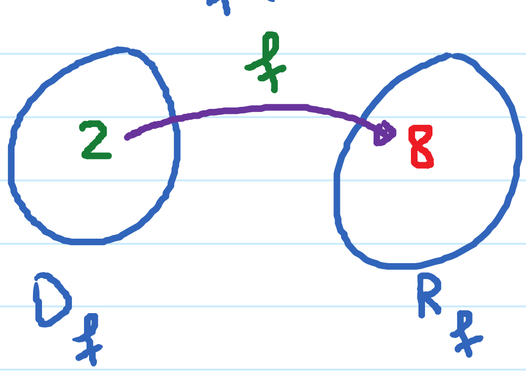
So, $g(f(x)) = x$.

This process verifies that f and g are inverse functions of one another. (In math notation, we can write:

$g = f^{-1}$ and $f = g^{-1}$)

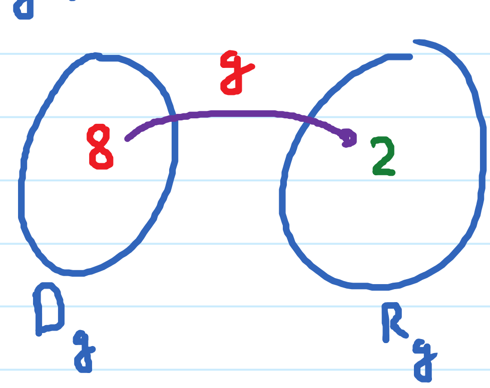
Note: $f(x) = 3x + 2$

$f(2) = 8$



$g(x) = \frac{x-2}{3}$

$g(8) = 2$



So, f undoes what g does and vice versa.

E.g. Given $h(x) = 4x - 7$ and $j(x) = \frac{x+7}{4}$.

Verify that h and j are inverse functions of one another.

$$h(j(x)) = \cancel{4}\left(\frac{x+7}{\cancel{4}}\right) - 7 = x + \cancel{7} \cancel{-7} = x$$

$$j(h(x)) = \frac{(4x \cancel{-7}) + \cancel{7}}{4} = \frac{\cancel{4}x}{\cancel{4}} = x.$$

Obj 2: Find the inverse of a function.

E.g. Find the inverse of $f(x) = 2x + 7$

Step 1: Replace the notation $f(x)$ with y

$$\boxed{y} = 2\boxed{x} + 7$$

Step 2: Interchange x and y in the above equation

$$x = 2y + 7$$

Step 3: Solve for y by itself

$$x - 7 = 2y \quad \text{(Subtract 7 from both sides)}$$

$$\frac{x-7}{2} = y \quad \text{(Divide both sides by 2)}$$

$$y = \frac{x-7}{2}$$

Step 4: Replace y by the notation $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \frac{x-7}{2}} \longleftarrow \text{Answer.}$$

E.g. Find the inverse of $f(x) = 4x^3 - 1$.

Step 1: Replace $f(x)$ with y

$$y = 4x^3 - 1$$

Step 2: Interchange x and y

$$x = 4y^3 - 1$$

Step 3: Solve for y by itself

$$x + 1 = 4y^3 \quad \text{(add 1 to both sides)}$$

$$\frac{x+1}{4} = y^3 \quad \text{(divide both sides by 4)}$$

$$\sqrt[3]{\frac{x+1}{4}} = y \quad \text{(take cube root of both sides)}$$

$$y = \sqrt[3]{\frac{x+1}{4}}$$

Step 4: Replace y with $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}}$$

Answer (this is the inverse function of f)

E.g. Find the inverse of $f(x) = \sqrt[3]{x} + 1$.

Step 1: $y = \sqrt[3]{x} + 1$ (Replace $f(x)$ with y)

Step 2: $x = \sqrt[3]{y} + 1$ (Interchange x and y)

Step 3: Solve for y:

$$x - 1 = \sqrt[3]{y}$$

$$(x-1)^3 = y \quad \text{(Cube both sides)}$$

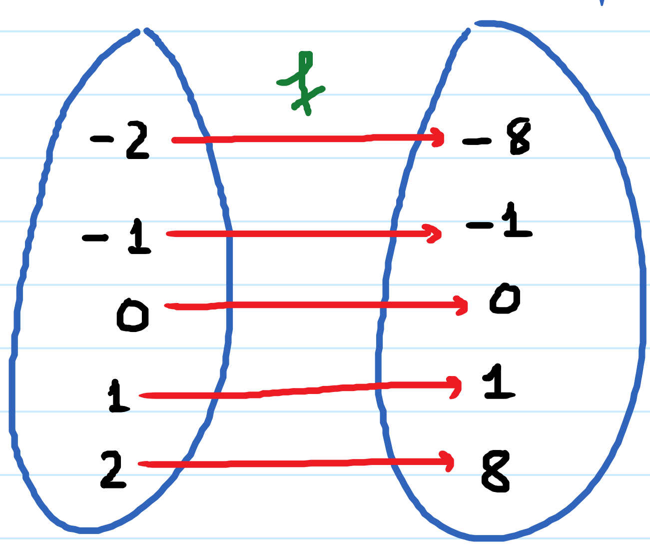
$$y = (x-1)^3$$

Step 4: $\boxed{f^{-1}(x) = (x-1)^3}$

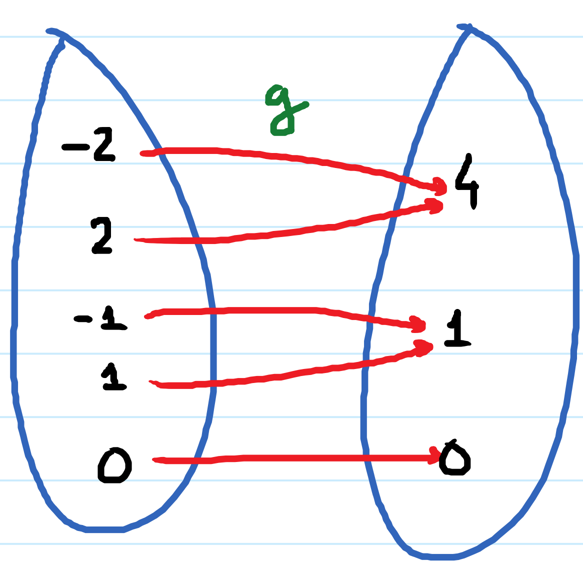
Obj 3: Horizontal line test and One-to-One functions

Note that not all functions have inverse functions.

The type of functions that have inverse functions is called one-to-one functions



One-to-one function

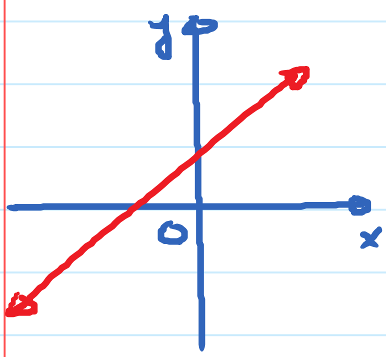


NOT a one-to-one function

Horizontal line test

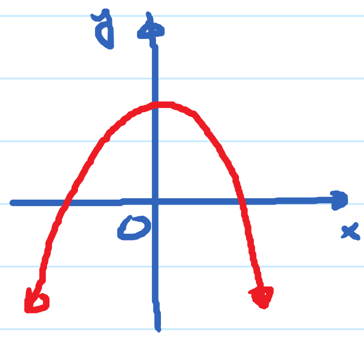
A function f is one-to-one (or has an inverse) if there is No horizontal line that intersects the graph of f at more than one point.

E.g.

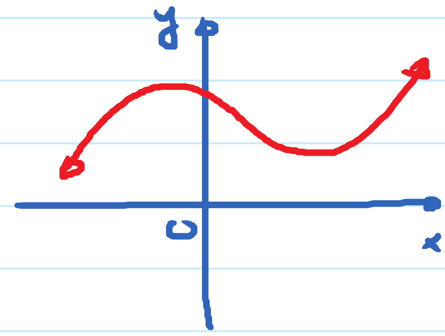


One-to-one

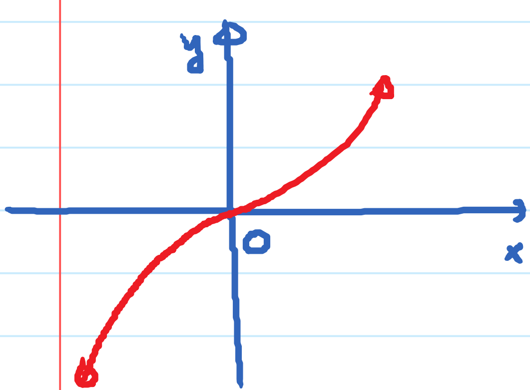
(has inverse)



Not one-to-one

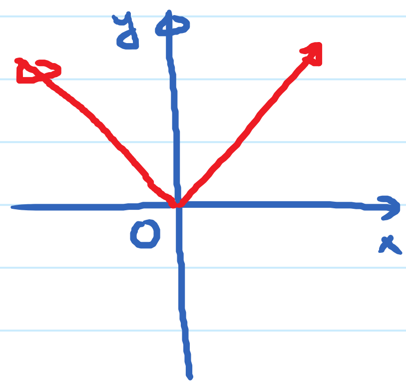


Not one-to-one

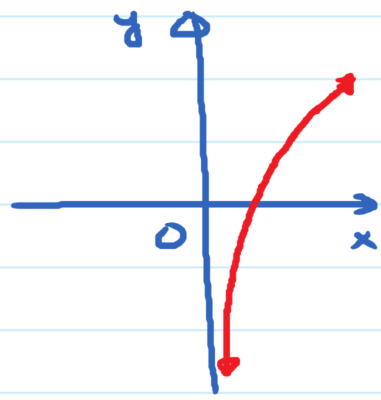


one-to-one

(has inverse)



Not one-to-one



one-to-one

(has inverse)