3.1. Quadratic functions.

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Objective 1: Characteristics of graphs of quadratic functions.

A quadratic function is a function of the form

$$f(x) = ax^2 + bx + c$$

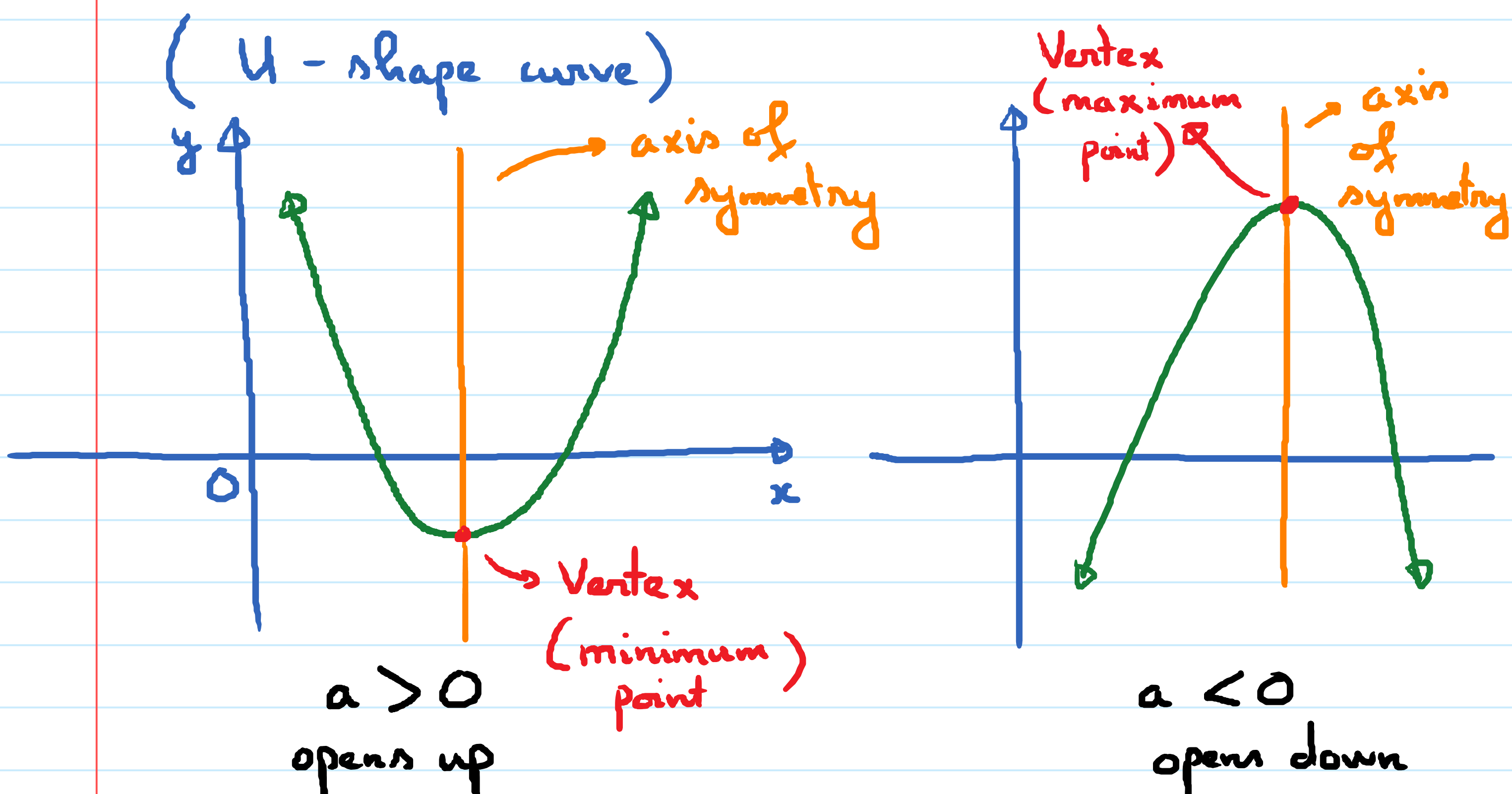
where a, b, c are real numbers and $a \neq 0$.

E.g. $f(x) = 4x^2 - 16x + 1000$

$a = 4$; $b = -16$; $c = 1000$

This is a quadratic function.

* The graph of any quadratic function is a parabola. (U-shape curve)



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Objective 2: Graph quadratic functions of the form $f(x) = ax^2 + bx + c$

* Vertex formula:

$$\boxed{x_{vertex} = -\frac{b}{2a}} \qquad \boxed{y_{vertex} = f\left(-\frac{b}{2a}\right)}$$

(We plug the value of x_{vertex} into the function)

E.g. Find the vertex of the parabola defined by the given quadratic function.

(a) $f(x) = 2x^2 - 8x + 3$

(b) $g(x) = -x^2 - 2x + 8$

Solution:

(a) $x_{vertex} = -\frac{b}{2a} = -\frac{(-8)}{2(2)} = \frac{8}{4} = 2$

$y_{vertex} = f(2) = 2(2)^2 - 8(2) + 3$

$= -5$

→ Vertex: $(2, -5)$

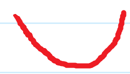
(b) $x_{vertex} = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$

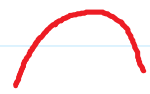
$y_{vertex} = f(-1) = -(-1)^2 - 2(-1) + 8 = 9$

$\boxed{\text{Vertex}: (-1, 9)}$

* Process for graphing $f(x) = ax^2 + bx + c$.

Step 1: Determine whether the parabola opens up or opens down.

$a > 0$: parabola opens up. 

$a < 0$: parabola opens down. 

Step 2: Find the vertex.

Vertex formula: $\left(\underbrace{-\frac{b}{2a}}_{x \text{ vertex}}, \underbrace{f\left(-\frac{b}{2a}\right)}_{y \text{ vertex}}\right)$

Step 3: Find 2 more points on the graph (in addition to vertex)

* Method 1: Find the y-intercept.

The y intercept is the point where $x = 0$.

So: y-intercept is : $\boxed{(0, c)}$

(because when $x = 0$, $f(0) = c$)

To find another point, we can reflect the y-intercept across the axis of symmetry.

(axis of symmetry: $x = -\frac{b}{2a}$)

* **Method 2:** Find x-intercepts by solving the equation $f(x) = 0$.
  (This works well if the formula for the function is easily factorable)
* **Method 3:** Make a T-table of (x,y) values and use symmetry

E.g. Graph the quadratic function:

$$f(x) = -x^2 - 2x + 1.$$

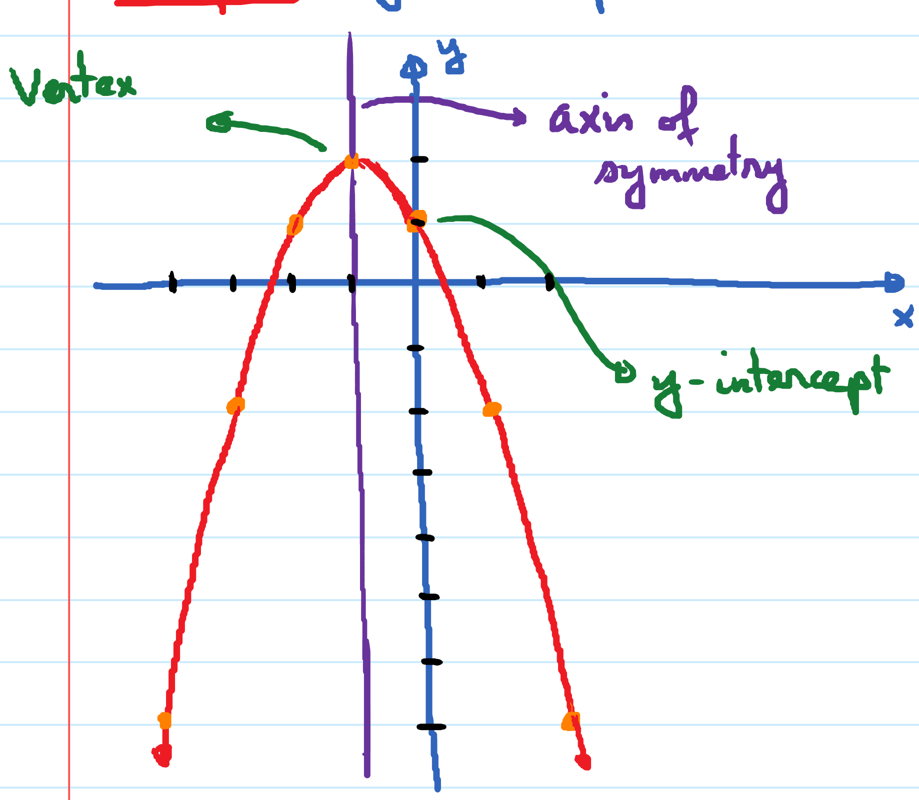
Step 1: $a = -1 < 0$. Opens down.

Step 2: Vertex. $x_{vertex} = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$

y vertex: $f(-1) = -(-1)^2 - 2(-1) + 1$
$= 2$

Vertex: $(-1, 2)$

Step 3: y-intercept: $(0, 1)$



x	$y = -x^2 - 2x + 1$
1	$-2 \to (1, -2)$
2	-7

Obj 3: Find minimum and maximum of a quadratic function.

Given $f(x) = ax^2 + bx + c$.

If $a > 0$, then f has a minimum point. The minimum point is the same as the vertex.

If $a < 0$, then f has a maximum point. The maximum point is the same as the vertex.

Note: If problem asks for max value or min value, it is the y of vertex that the problem asks for.

E.g. Given $f(x) = -3x^2 + 6x - 13$.

Q1: Does f have a max or a min?

Q2: Find the max or min value and where it occurs.

Q3: Find domain of f. Find range of f.

Solutions:

Q1: Max. opens down $(a = -3 < 0)$

Q2: $x_{\text{vertex}} = -\dfrac{b}{2a} = -\dfrac{6}{2(-3)} = 1$

$y_{\text{vertex}} = f(1) = -3(1)^2 + 6(1) - 13$
$= -10$.

Max value of $f = -10$

and it occurs where $x = 1$

Q3: Domain of $f = (-\infty, \infty)$ (No restrictions)

Range of $f = (-\infty, -10]$