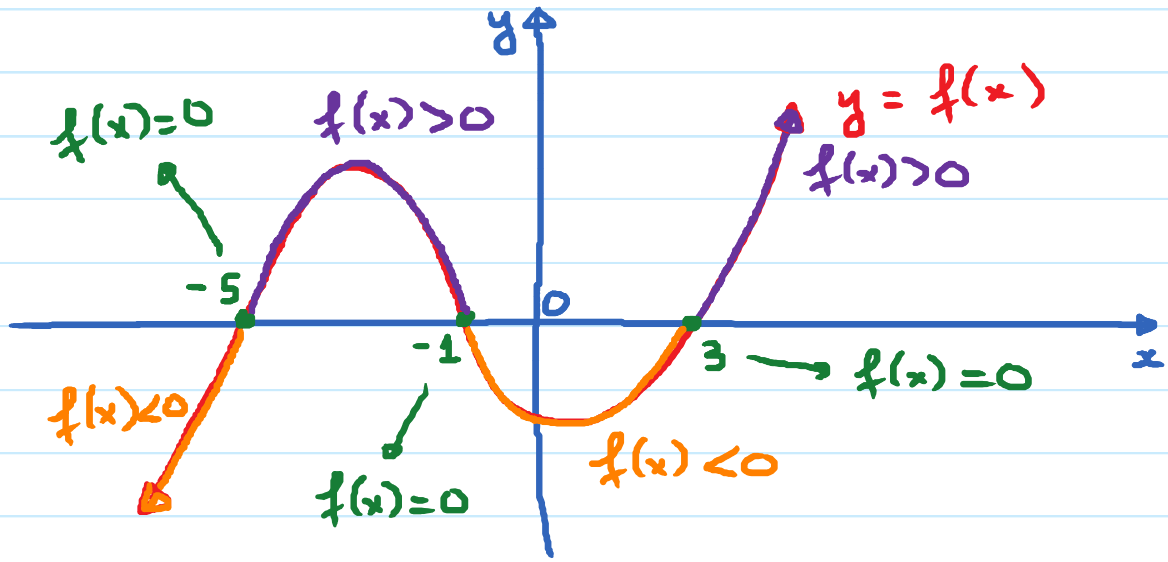
3.6. Polynomial and Rational Inequalities

Wednesday, November 13, 2019 12:37 PM

What does it mean to solve an inequality of the form $f(x) > 0$ or $f(x) < 0$ or $f(x) \geq 0$ or $f(x) \leq 0$?



* Before, when we solve the equation $f(x) = 0$, we are looking for the x-intercepts.

So, the solution set to the equation $f(x) = 0$ is:

$x = -5$; $x = -1$; $x = 3$

* When we solve the inequality $f(x) > 0$, we are looking for intervals over which the graph of

$f(x)$ is above the x-axis $(y>0)$

Solution set to the inequality $f(x)>0$ is the intervals: $(-5,-1)\cup(3,\infty)$

Similarly, solution set to $f(x)<0$ is:

$$(-\infty,-5)\cup(-1,3).$$

Solution set to $f(x)\geq 0$ is:

$$[-5,-1]\cup[3,\infty)$$

Solution set to $f(x)\leq 0$ is:

$$(-\infty,-5]\cup[-1,3]$$

<u>Obj 1:</u> Solve Polynomial Inequalities.

<u>E.g.</u> Solve the inequality; graph the solution set on a number line: $\boxed{x^2-x>20}$

Step 1: Express the given inequality in the form

$f(x) > 0$ or $f(x) < 0$ (or $f(x) \geq 0$ or $f(x) \leq 0$)

$$x^2 - x > 20$$

$$\underbrace{\boxed{x^2 - x - 20}}_{f(x)} > 0$$ (Subtract 20 from both sides)

Step 2: Find the "special" values of x by solving the equation $f(x) = 0$

$$x^2 - x - 20 = 0$$ (Set $f(x) = 0$)

$$(x - 5)(x + 4) = 0$$ (Factor)

$$x - 5 = 0 \quad ; \quad x + 4 = 0$$

$$\underbrace{\boxed{x = 5} \quad ; \quad \boxed{x = -4}}_{\text{"special" Values of } x.}$$

Step 3: Locate these special values on a number line and separate the line into intervals.

These special values divide the number line into 3 intervals : $(-\infty,-4)$; $(-4,5)$; $(5,\infty)$

Since we are solving the inequality $f(x)>0$, we just need to determine the interval(s) for which $f(x)>0$ (positive)

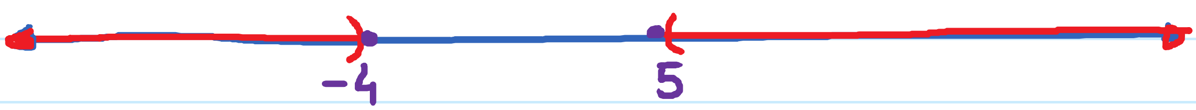
Step 4: Choose one test value within each interval and plug that test value into f.

Interval	$(-\infty,-4)$	$(-4,5)$	$(5,\infty)$
Test Value	-5	0	6
Substitute into $f(x)=x^2-x-20$	10	-20	10
	$f(x)>0$	$f(x)<0$	$f(x)>0$

Step 5: Write the solution set by selecting the interval(s) that satisfy the inequality.

Since we are solving $f(x) > 0$, the solution set is: $\boxed{(-\infty, -4) \cup (5, \infty)}$

Graph on a number line



Note: If we were solving $f(x) \geq 0$, solution set will be: $(-\infty, -4] \cup [5, \infty)$

E.g. Solve and graph the solution set of

$$x^3 + 3x^2 \leq x + 3.$$

Solution:

1 $\underbrace{\boxed{x^3 + 3x^2 - x - 3}}_{f(x)} \leq 0$ (Get 0 on one side)