

② Set : $x^3 + 3x^2 - x - 3 = 0$ (Set $f(x) = 0$)

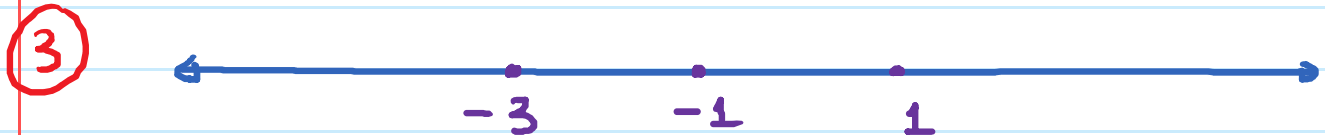
$$x^2(x+3) - (x+3) = 0$$

$$(x+3)(x^2-1) = 0$$

$$x+3=0 \quad ; \quad x^2-1=0$$

$$x = -3 \quad ; \quad x^2 = 1 \rightarrow x = \pm 1$$

Special values: $-3, -1, 1$.

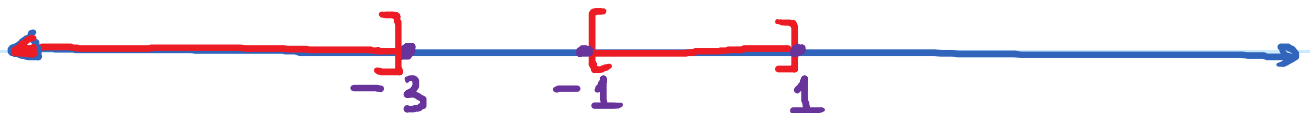


Intervals to consider: $(-\infty, -3)$, $(-3, -1)$, $(-1, 1)$,
 $(1, \infty)$

④ Intervals	$(-\infty, -3)$	$(-3, -1)$	$(-1, 1)$	$(1, \infty)$
Test value	-4	-2	0	2
Substitute into $f(x) = x^3 + 3x^2 - x - 3$	-15	3	-3	15
Conclusion	$f(x) < 0$	$f(x) > 0$	$f(x) < 0$	$f(x) > 0$

⑤ We are solving $f(x) \leq 0$.

So the solution set is : $(-\infty, -3] \cup [-1, 1]$



Obj 2: Solve Rational Inequalities.

E.g. Solve and graph the solution set of the inequality : $\frac{x+1}{x+3} \geq 2$

Solution:

Step 1: Express the inequality so that one side is 0 and the other side to be a single "fraction".

$$\frac{x+1}{x+3} - 2 \geq 0$$

Need to combine into a single fraction

$$\text{LCD} = x+3$$

$$\frac{x+1}{x+3} - \frac{2 \cdot (x+3)}{1 \cdot (x+3)} \geq 0$$

$$\frac{x+1}{x+3} - \frac{2x+6}{x+3} \geq 0$$

$$\frac{x+1 - (2x+6)}{x+3} \geq 0$$

$$\frac{x+1 - 2x - 6}{x+3} \geq 0$$

$$\boxed{\frac{-x-5}{x+3}} \geq 0$$

$f(x)$

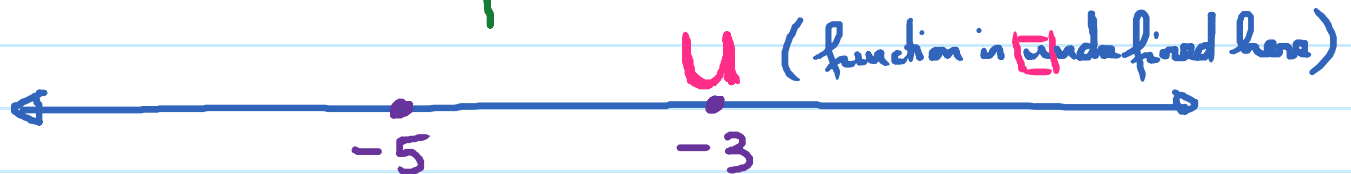
Step 2: Find the "special" values of x by setting both numerator and denominator of $f(x)$ equal to 0

$$\begin{array}{ll} -x-5=0 & ; \quad x+3=0 \\ (\text{numerator}=0) & (\text{denominator}=0) \end{array}$$

$$\boxed{x = -5} \quad ; \quad \boxed{x = -3}$$

Special values.

Step 3: Locate these special values on a number line and separate the line into intervals:



Intervals: $(-\infty, -5)$; $(-5, -3)$; $(-3, \infty)$

Step 4: Test Values.

Intervals:	$(-\infty, -5)$	$(-5, -3)$	$(-3, \infty)$
Test Value	-6	-4	0
Substitute these values into: $f(x) = \frac{-x-5}{x+3}$	$\frac{-(-6)-5}{-6+3}$ $= \frac{1}{-3} = -\frac{1}{3}$	$\frac{-(-4)-5}{-4+3}$ $= \frac{-1}{-1} = 1$	$\frac{-(0)-5}{0+3}$ $= \frac{-5}{3}$
Conclusion	$f(x) < 0$	$f(x) > 0$	$f(x) < 0$

Step 5: Select the correct intervals.

We are solving $f(x) \geq 0$, the solution

set is : $[-5, -3)$

we cannot include -3
because f is undefined
when $x = -3$ (denom. $= 0$)

