1.5. Quadratic Equations

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Obj 1: Definition of a quadratic equation

E.g. $\boxed{x^2 - 7x + 10 = 0}$

A quadratic equation is an equation of the form

$$ax^2 + bx + c = 0$$

where a, b, c are real numbers and $a \neq 0$

In the above equation : $a = 1$; $b = -7$; $c = 10$

Obj 2: Solve quadratic equations by factoring

$x^2 - 7x + 10 = 0$ → this form is hard to solve for x

$\underbrace{(x-2)\cdot(x-5) = 0}$ → this form is easier to solve for x

To solve this : we set

$x - 2 = 0$ or $x - 5 = 0$

$x = 2$ or $x = 5$

Claim: $(x-2)(x-5) = \boxed{x^2 - 7x + 10}$

Why? $(x-2)(x-5)$

$= x^2 \underbrace{- 5x - 2x}_{\text{like terms}} + 10$

$= \boxed{x^2 - 7x + 10}$

* To solve a quadratic equation by factoring is to go from the standard form ($ax^2+bx+c=0$) to the factored form and solve the factored form by setting each factor equal to zero.

Eg. Solve quadratic equations by factoring

(a) $4x^2 - 2x = 0$

$2x(2x-1) = 0$ (Factor out the common factor $2x$)

$2x = 0$ or $2x - 1 = 0$ (Set each factor equal to zero)

$\boxed{x = 0}$ $\boxed{x = \frac{1}{2}}$

Solution set: $\boxed{\left\{0, \frac{1}{2}\right\}}$

(b) $2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$ (Right-hand side must be zero when solve by factoring)

$(2x - 1)(x + 4) = 0$ (Factor)

$2x - 1 = 0$ or $x + 4 = 0$ (Set each factor equal to zero)

$x = \frac{1}{2}$ $x = -4$

Solution set : $\boxed{\left\{ \frac{1}{2}, -4 \right\}}$

(c) $2x^2 + x = 1$

$2x^2 + x - 1 = 0$ (Right hand side $= 0$)

$(2x - 1)(x + 1) = 0$ (Factor)

$2x - 1 = 0$ or $x + 1 = 0$

$x = \frac{1}{2}$ or $x = -1$. Solution set $\boxed{\left\{ \frac{1}{2}, -1 \right\}}$

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Obj 3: Solve a quadratic equation by the Square Root Property

E.g. $x^2 = 4 \longrightarrow x = \pm\sqrt{4} = \pm 2$

In general, the square root property says that if u is an expression and d is a number and we have $u^2 = d$

Then $u = \sqrt{d}$ or $u = -\sqrt{d}$

We can write this in an equivalent way:

$$\boxed{u^2 = d \longrightarrow u = \pm\sqrt{d}}$$

E.g. Solve quadratic equations by the Square Root Property

(a) $3x^2 - 15 = 0$

Isolate x^2: $3x^2 = 15 \longrightarrow x^2 = 5$

Square root property: $x = \pm\sqrt{5}$

Solution set: $\{\sqrt{5}, -\sqrt{5}\}$

Note: Before you can apply the Square Root Property, a Squared Expression must be isolated on one side of the equation.

(b) $5x^2 + 45 = 0$

$\rightarrow 5x^2 = -45 \rightarrow \boxed{x^2 = -9}$

Recall: Imaginary unit $i^2 = -1$

By Square Root Property

$x = \pm\sqrt{-9} = \pm\sqrt{i^2 \cdot 9}$

$= \pm\sqrt{i^2} \cdot \sqrt{9} = \pm i \cdot 3$

or write as $x = \pm 3i$

Solution set: $\{3i, -3i\}$ imaginary numbers

c) $(x-2)^2 = 6$

By the Square Root Property

$$x - 2 = \pm\sqrt{6}$$

$$x = \pm\sqrt{6} + 2$$

or we can write as $x = 2 \pm \sqrt{6}$

Solution set: $\boxed{\{2+\sqrt{6},\ 2-\sqrt{6}\}}$

Obj 4: Solving Quadratic Equations by Using the Quadratic Formula.

The solutions of a quadratic equation

$$ax^2 + bx + c = 0 \quad ; \quad a \neq 0$$

are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is the quadratic formula

E.g. Solve the equation $8x^2 + 2x - 1 = 0$

$a = 8$; $b = 2$; $c = -1$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot(8)\cdot(-1)}}{2(8)}$$

$$= \frac{-2 \pm \sqrt{36}}{16} = \frac{-2 \pm 6}{16}$$

So, $x = \dfrac{-2 + 6}{16} = \dfrac{4}{16} = \dfrac{1}{4}$

or $x = \dfrac{-2 - 6}{16} = \dfrac{-8}{16} = -\dfrac{1}{2}$

Solution set $\boxed{\left\{ \frac{1}{4}, -\frac{1}{2} \right\}}$

E.g. Solve $2x^2 - 6x + 1 = 0$

$a = 2$; $b = -6$; $c = 1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4\cdot(2)\cdot(1)}}{2\cdot(2)}$$

$$= \frac{6 \pm \sqrt{28}}{4} = \frac{6 \pm \sqrt{4\cdot 7}}{4}$$

$$x = \frac{6 \pm \sqrt{4}\cdot\sqrt{7}}{4} = \frac{6 \pm 2\sqrt{7}}{4}$$

$$= \frac{\cancel{2}(3 \pm \sqrt{7})}{\cancel{4}\,2} = \frac{3 \pm \sqrt{7}}{2}$$

Solution set: $\boxed{\left\{ \frac{3+\sqrt{7}}{2} ; \frac{3-\sqrt{7}}{2} \right\}}$

E.g. Non-real Solutions

Solve $x^2 - 2x + 2 = 0$

$a = 1$; $b = -2$; $c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot(1)\cdot(2)}}{2\cdot(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm \sqrt{i^2\cdot 4}}{2}$$

$$= \frac{2 \pm \sqrt{i^2}\cdot\sqrt{4}}{2} = \frac{2 \pm i\cdot 2}{2}$$

$$= \frac{\cancel{2}(1 \pm i)}{\cancel{2}} = 1 \pm i$$

Solution set: $\boxed{\{1+i, 1-i\}}$