2.2. More on Functions and their Graphs

Objective 1: Even and Odd Functions

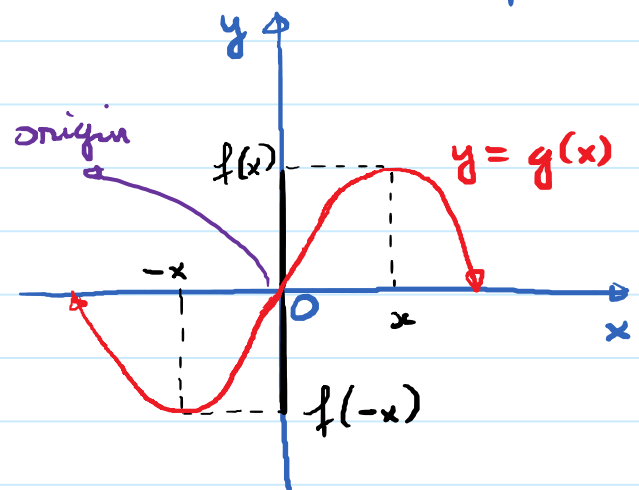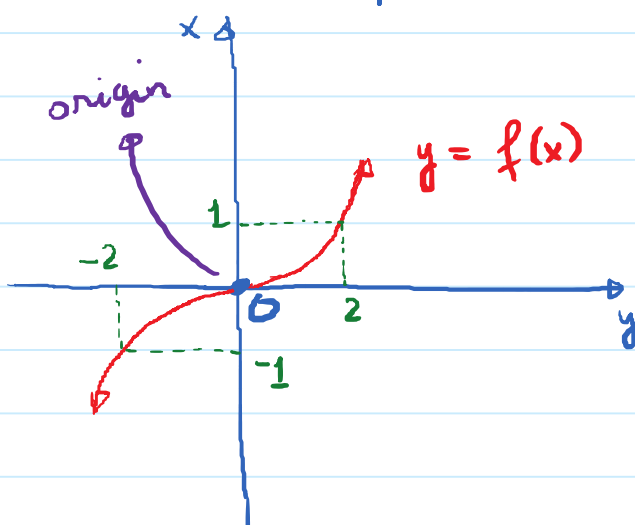**Definition:** A function whose graph is symmetric with respect to the y-axis is called an EVEN function.

EVEN function
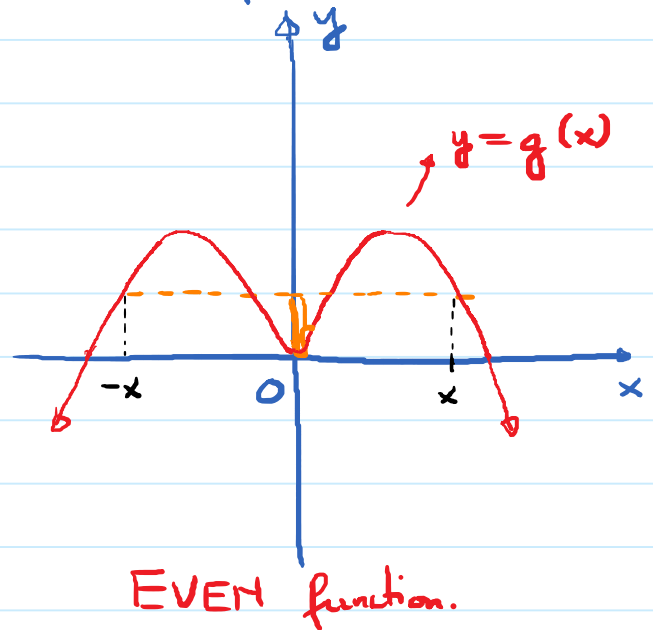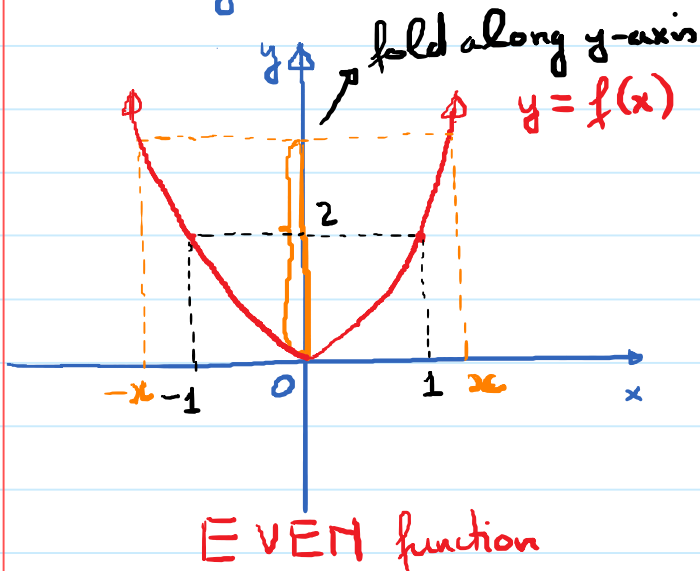
EVEN function.

NOTE: $f(-x) = f(x)$

Definition: A function whose graph is symmetric with respect to the origin is called an ODD function.

ODD function

ODD function.

NOTE: $f(-x) = -f(x)$

* How to determine whether a function $y = f(x)$ is odd or even (or neither) given the equation for $f(x)$:

Step 1: Find $f(-x)$ by replacing x by $-x$ in the equation for $f(x)$

Step 2. Simplify and compare the equation for $f(x)$ and the equation for $f(-x)$. If $f(-x) = f(x)$, then the function is even. If $f(-x) = -f(x)$ then the function is odd. If neither equality holds, then the function is neither odd nor even.

E.g. Determine whether the given function is odd or even or neither.

ⓐ $\boxed{f(x) = x^4 - 2x^2.}$

Step 1: Find $f(-x)$:
$$f(-x) = (-x)^4 - 2(-x)^2$$

Step 2: Simplify:
$$f(-x) = (-x)^4 - 2(-x)^2$$
$\Big)$ Simplify
$$\boxed{f(-x) = x^4 - 2x^2}$$

Same

Compare $f(-x)$ and $f(x)$: $f(-x) = f(x)$.

Conclusion: f is an even function.

(b) $\boxed{f(x) = x^3 - 6x}$

opposite

Step 1: Find $f(-x)$

$$f(-x) = (-x)^3 - 6(-x)$$

Step 2: Simplify

$$f(-x) = (-x)^3 - 6(-x)$$

$$\boxed{f(-x) = -x^3 + 6x = \ominus \left(\boxed{x^3 - 6x}\right)}$$

original

Compare $f(x)$ and $f(-x)$: $f(-x) = -f(x)$.

Conclusion: f is an odd function.

(c) $\boxed{f(x) = x^2 + 2x + 1}$

Not same

Not opposite

$$f(-x) = (-x)^2 + 2(-x) + 1$$

Simplify: $\boxed{f(-x) = x^2 - 2x + 1}$

Conclusion: f is Neither odd nor even.

Objective: Piecewise Functions.

condition for x

E.g.

condition for x

$$f(x) = \begin{cases} 3x + 5 & \boxed{\text{if } x < 0} \\ \\ 4x + 7 & \boxed{\text{if } x \geq 0} \end{cases}$$

(a) $f(\boxed{-2})$ -2 is less than $0 \rightarrow$ use first formula

$$f(-2) = 3(-2) + 5 = -6 + 5 = -1$$

(Replace x by -2 in first formula)

So, $f(-2) = -1$.

(b) $f(\boxed{3})$ 3 is greater than 0 → use second formula

$$f(3) = 4(3) + 7 = 12 + 7 = 19.$$

(Replace x by 3 in second formula)

So, $f(3) = 19$.

(c) $f(\boxed{0})$ equal 0 → second formula

So, $f(0) = 4(0) + 7 = 7$.

E.g. Graph the function:

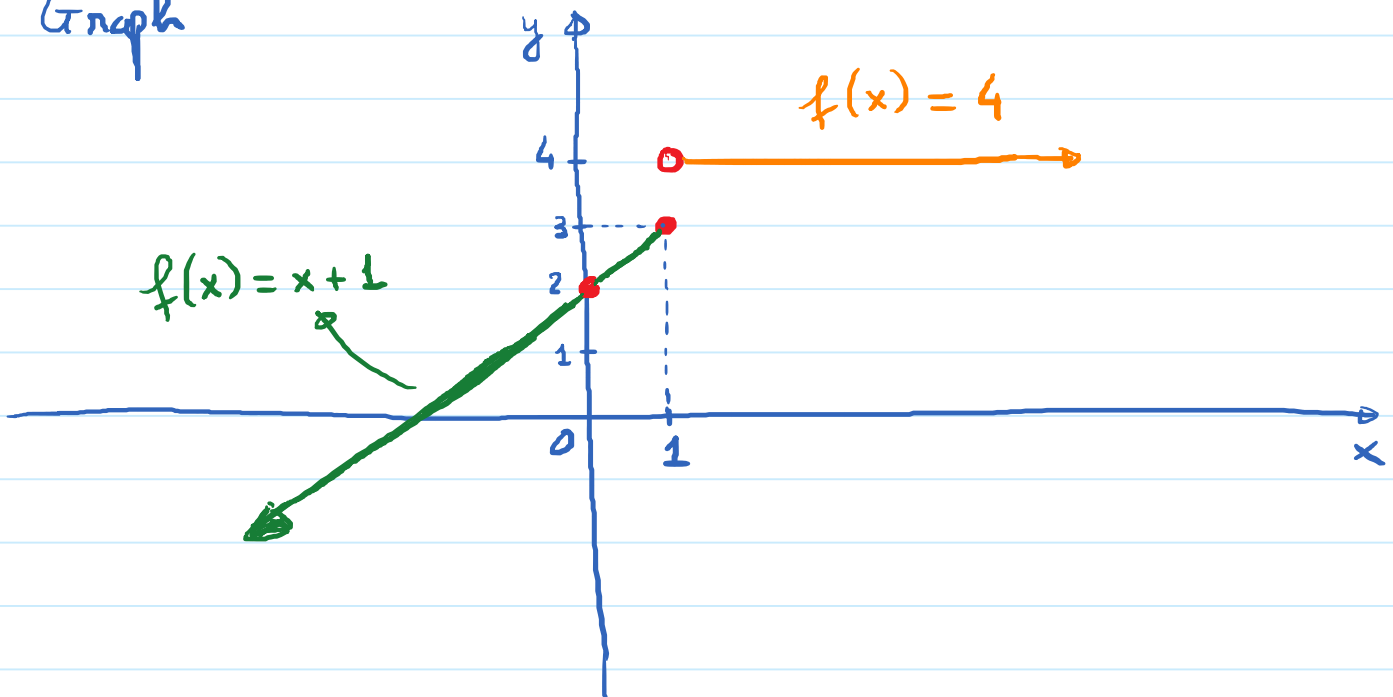$$f(x) = \begin{cases} x + 2 & \text{if } x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$$

Graph each formula:

1st formula: $f(x) = x + 2$ (if $x \leq 1$)

x	$f(x) = x+2$	ordered pair
1	3	(1, 3)
0	2	(0, 2)

2nd formula: $f(x) = 4$ (if $x > 1$)

Graph

$f(x) = 4$

$f(x) = x + 1$

Ex: Evaluate at the given value:

(a) $g(x) = \begin{cases} x + 3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$

$g(0) = 3$ $g(-6) = 3$ $g(-3) = 0$

(b) $h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

$h(5) = 8$ $h(0) = 3$ $h(3) = 6$