Lecture 1 - Area between Curves

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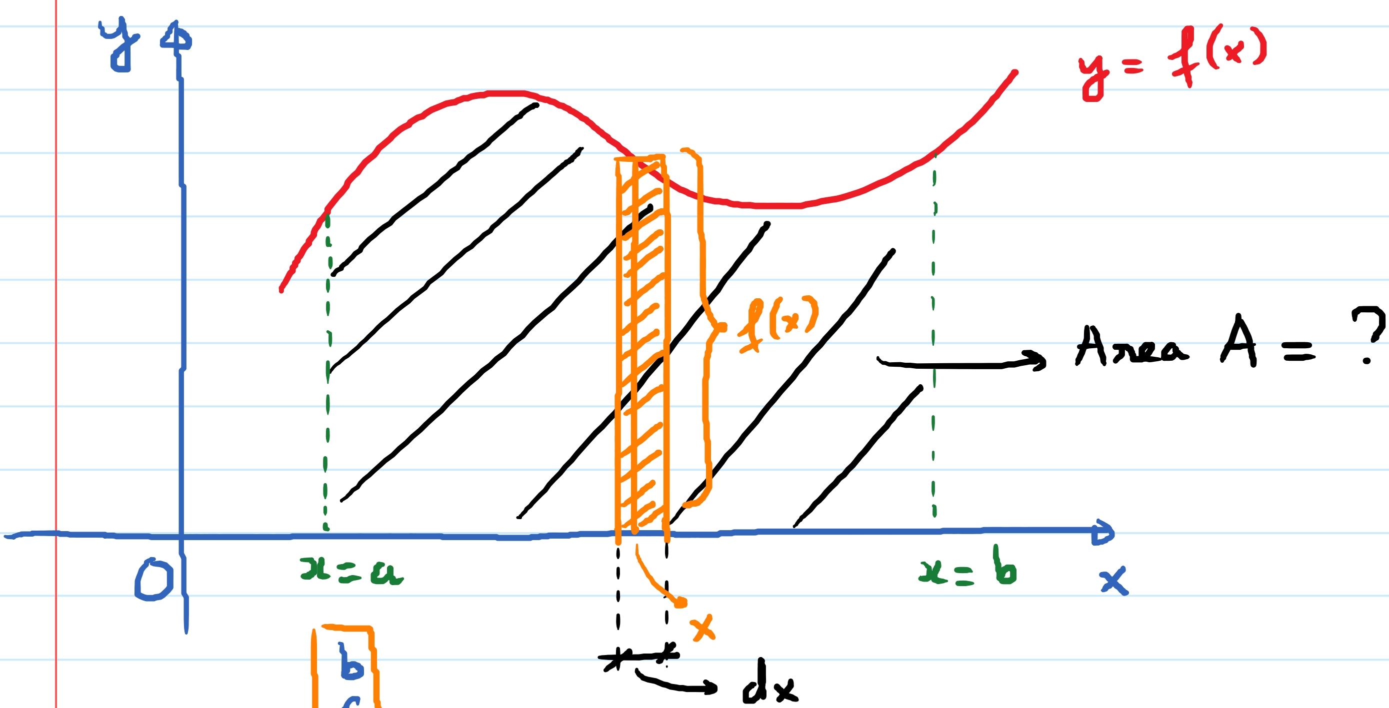
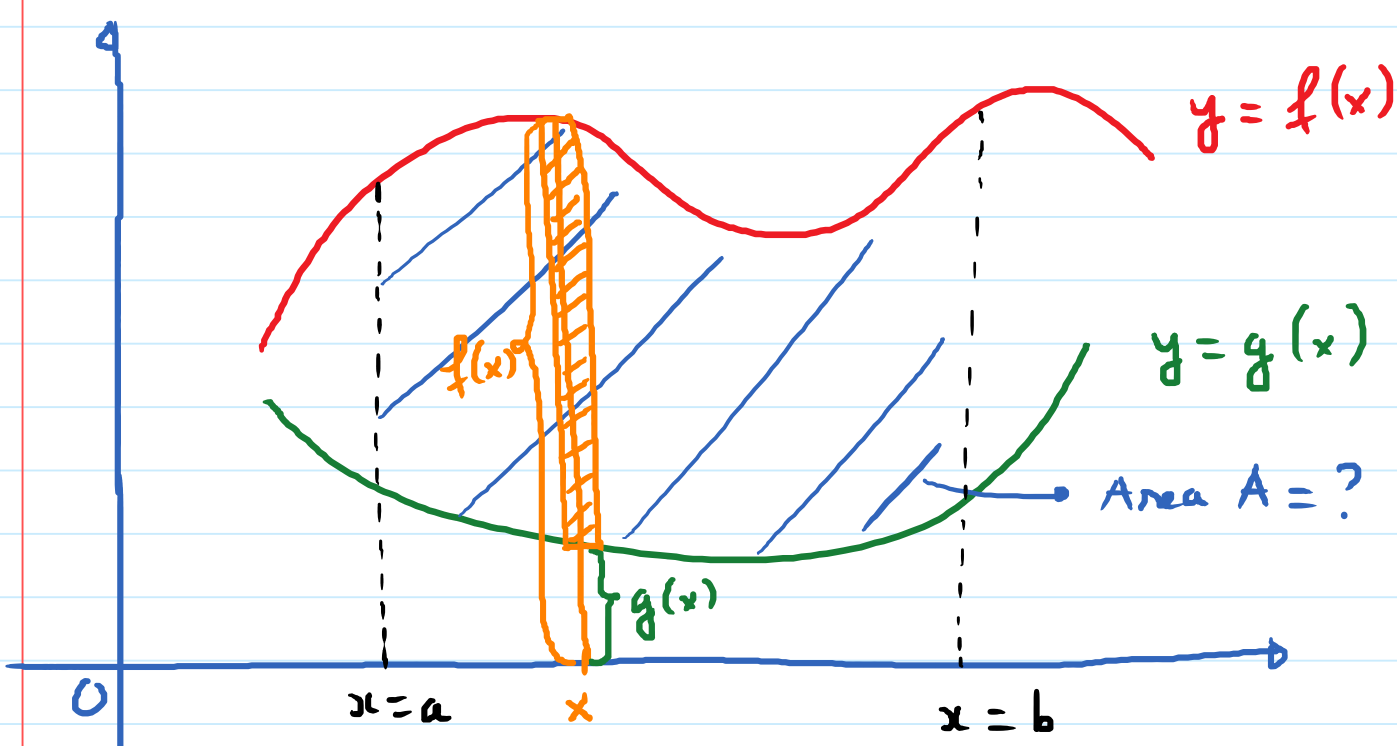
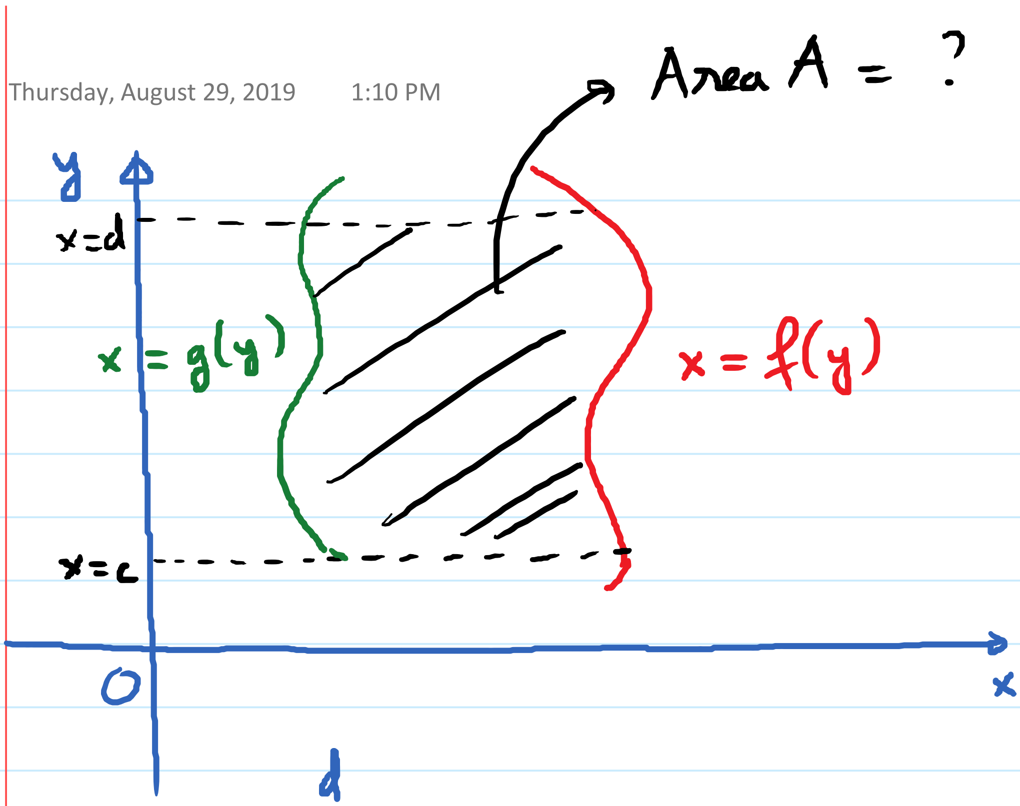


$$A = \boxed{\int_a^b} \underbrace{f(x)}_{\text{height of small rectangle}} \underbrace{dx}_{\text{width of a small rectangle}}$$



$$A = \int_a^b \Big[\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}} \Big] dx$$

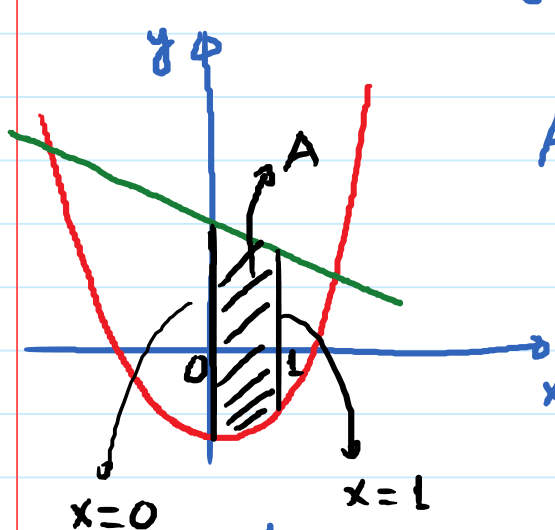


$$A = \int_c^d \Big[\underbrace{f(y)}_{\text{rightmost}} - \underbrace{g(y)}_{\text{leftmost}}\Big]\,dy$$

E.g. 1. $y = x^2 - 1$; $y = -x + 2$

$x = 0$; $x = 1$

Find area of region.



$$A = \int_0^1 \Big[\underbrace{(-x+2)}_{\text{top}} - \underbrace{(x^2-1)}_{\text{bottom}}\Big]\,dx$$

$$A = \int_0^1 \left[-x + 2 - x^2 + 1\right] dx = \int_0^1 \left[-x^2 - x + 3\right] dx$$

$$= \left(-\frac{x^3}{3} - \frac{x^2}{2} + 3x\right)\Bigg|_0^1 = -\frac{1}{3} - \frac{1}{2} + 3 = \boxed{\frac{13}{6}}$$

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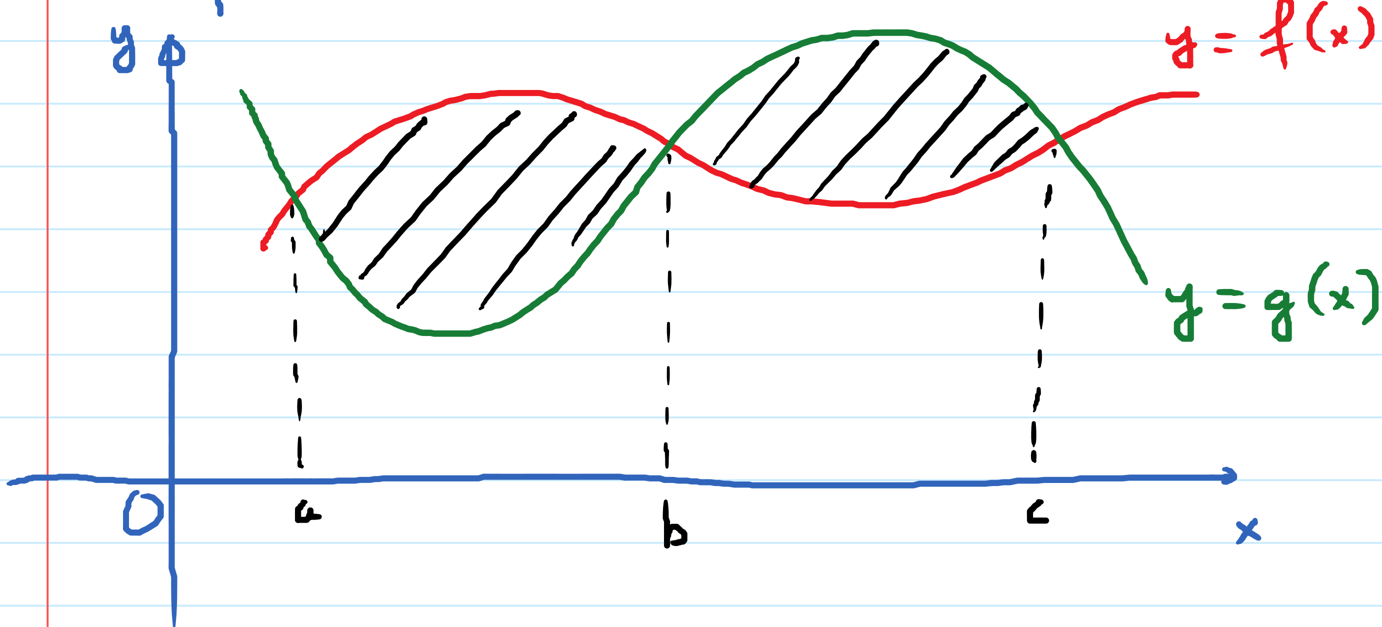
<u>Note:</u> Case where the 2 curves intersect at more than 2 points.



<u>Step 1:</u> Find points of intersection by setting $f(x) = g(x)$ and solve for x

<u>Step 2:</u> $$A = \int_a^b \Big(\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}}\Big)\,dx + \int_b^c \Big(\underbrace{g(x)}_{\text{top}} - \underbrace{f(x)}_{\text{bottom}}\Big)\,dx$$

<u>E.g. 2.</u>

Points of Intersection:

$$x^3 - 3x^2 + 3x = x^2$$

$$x^3 - 4x^2 + 3x = 0$$

$$x(x^2 - 4x + 3) = 0$$

$$x(x-1)(x-3) = 0$$

$$\boxed{x = 0;\quad x = 1,\quad x = 3}$$

x-coord of points of int.

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Step 2: Area

$$A = \int_0^1 \left(x^3 - 3x^2 + 3x - x^2\right) dx + \int_1^3 \left[x^2 - \left(x^3 - 3x^2 + 3x\right)\right] dx$$

$$= \int_0^1 \left(x^3 - 4x^2 + 3x\right) dx + \int_1^3 \left(-x^3 + 4x^2 - 3x\right) dx$$

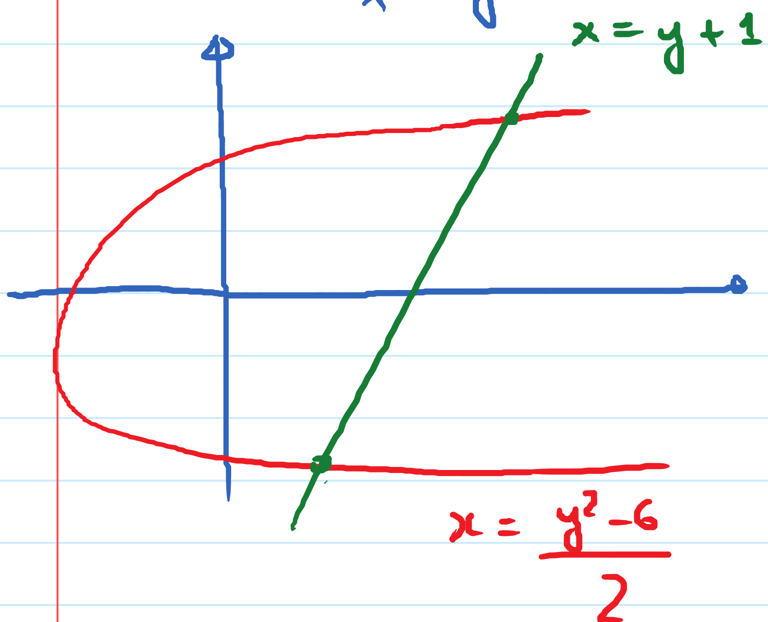
$$= \left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right)\Big|_0^1 + \left(-\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2}\right)\Big|_1^3$$

$$= \frac{5}{12} + \frac{8}{3} = \boxed{\frac{37}{12}}$$

E.g. 3 $y = x - 1 \longrightarrow x = y + 1$

$y^2 = 2x + 6 \longrightarrow x = \frac{y^2 - 6}{2}$

Area of region bounded by these 2 curves.



$$y + 1 = \frac{y^2 - 6}{2}$$

$$2y + 2 = y^2 - 6$$

$$y^2 - 2y - 8 = 0$$

$$(y - 4)(y + 2) = 0$$

$$y = 4; \quad y = -2.$$

$$A = \int_{-2}^{4} \Big(\underbrace{(y+1)}_{\text{Rightmost}} - \underbrace{\left(\frac{y^2-6}{2} \right)}_{\text{leftmost}} \Big)\, dy$$

$$= \int_{-2}^{4} \left(-\frac{y^2}{2} + y + 4 \right) dy$$

$$= \boxed{18}$$

Eg. 4

Note: $\boxed{\int e^{Kt}\,dt = \frac{e^{Kt}}{K} + C}$

$$\int_{0}^{10} \left(2000e^{0.02t} - 800e^{0.01t} \right) dt$$

$$= 2000 \int_{0}^{10} e^{0.02t}\, dt - 800 \int_{0}^{10} e^{0.01t}\, dt$$

$$= 2000 \cdot \left. \frac{e^{0.02t}}{0.02} \right|_{0}^{10} - 800 \left. \frac{e^{0.01t}}{0.01} \right|_{0}^{10}$$

$$= 100000\left(e^{0.2} - 1\right) - 80000\left(e^{0.1} - 1\right)$$

$$\simeq \boxed{13726.6}$$