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Lecture 0 : Review of Basic Integration Rules

$$\int k\, f(x)\,dx = k \int f(x)\,dx$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad ; \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

$$\int \csc^2(x)\,dx = -\cot(x) + C$$

$$\int \csc(x)\cot(x) = -\csc(x) + C$$

E.g. 1.

(1) $\int\left(x^{\frac{3}{2}}+2x+1\right)dx = \int x^{\boxed{\frac{3}{2}}}dx + 2\int x\,dx + \int 1\,dx$ ↗ n

$= \boxed{\frac{2}{5}x^{\frac{5}{2}} + x^2 + x + C}$

(2) $\int\left(2-\frac{3}{x^{10}}\right)dx = \int 2\,dx - 3\int\frac{1}{x^{10}}dx$

$= 2x - 3\int x^{-10}dx = 2x - 3\cdot\frac{x^{-9}}{-9} + C$

$= \boxed{2x + \frac{1}{3x^9} + C}$

(3) $\int(\sin x - 6\cos x)\,dx = \boxed{-\cos x - 6\sin x + C}$

(4) $\int\frac{5}{x}dx = 5\cdot\int\frac{1}{x}dx = \boxed{5\ln|x| + C}$

(5) $\int\frac{dx}{\sqrt{9-x^2}} = \boxed{\arcsin\left(\frac{x}{3}\right) + C}$

(6) $\int\frac{dx}{x^2+25} = \boxed{\frac{1}{5}\arctan\left(\frac{x}{5}\right) + C}$

E.g. 2.

① $\int (x+1)(3x-2)\,dx$

$$= \int (3x^2 + x - 2)\,dx = \boxed{x^3 + \frac{x^2}{2} - 2x + C}$$

② $\int \frac{x^4 - 3x^2 + 5}{x^4}\,dx$

$$= \int (x^4 - 3x^2 + 5) \cdot x^{-4}\,dx = \int (1 - 3x^{-2} + 5x^{-4})\,dx$$

$$= \boxed{x + 3x^{-1} - \frac{5}{3}x^{-3} + C}$$

Reminder: Fundamental Theorem of Calculus.

$$\boxed{\int_a^b f(x)\,dx = \underbrace{F(x)}_{\text{an antiderivative of } f(x)}\Big|_a^b = F(b) - F(a)}$$

To find a definite integral.

① Antiderivative

② Plug in upper and lower bound and subtract.

E.g. 2 ③ $\int_1^4 \frac{x-2}{\sqrt{x}}\,dx = \int_1^4 (x-2) \cdot x^{-\frac{1}{2}}\,dx$

$$= \int_1^4 \left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right) dx = \left(\frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}}\right)\Big|_1^4$$

$$= \left[\frac{2}{3}(4)^{\frac{3}{2}} - 4(4)^{\frac{1}{2}}\right] - \left[\frac{2}{3}(1)^{\frac{3}{2}} - 4(1)^{\frac{1}{2}}\right]$$

$$= \left(\frac{16}{3} - 8\right) - \left(\frac{2}{3} - 4\right) = \boxed{\frac{2}{3}}$$

④ $$\int_0^{\pi/4} \frac{1 - \sin^2 x}{\cos^2 x}\,dx \qquad \left(\sin^2 x + \cos^2 x = 1 \rightarrow 1 - \sin^2 x = \cos^2 x\right)$$

$$\int_0^{\pi/4} \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}}\,dx = \int_0^{\pi/4} 1\,dx = x\Big|_0^{\pi/4} = \boxed{\frac{\pi}{4}}$$

Review of u-sub.

E.g. 3 $$\int x \cdot \sqrt{x^2 + 2}\,dx$$

Let $\boxed{u = x^2 + 2}$ $\qquad \boxed{du = 2x\,dx} \longrightarrow dx = \frac{du}{2x}$

$$\int \cancel{x} \cdot \sqrt{u} \cdot \frac{du}{2\cancel{x}} = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{\frac{3}{2}} + C = \boxed{\frac{1}{3}\left(x^2 + 2\right)^{\frac{3}{2}} + C}$$

Table of basic integrals:

$$\int \tan(x)\,dx = \int \frac{\sin(x)\,dx}{\cos(x)} \longrightarrow \int \frac{-du}{u} = -\int \frac{du}{u}$$

$u = \cos(x)$. $\longrightarrow du = -\sin(x)\,dx$

$$= -\ln|u| + C$$

$$= -\ln|\cos(x)| + C$$

Eg.3

② $$\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$$

$x=0: \quad u = 2(0)+1 = 1$
$x=4: \quad u = 2(4)+1 = 9$

Let $u = 2x+1 \longrightarrow du = 2\,dx \longrightarrow dx = \frac{du}{2}$

$$\int_1^9 \frac{1}{\sqrt{u}}\cdot\frac{du}{2} = \frac{1}{2}\int_1^9 u^{-\frac{1}{2}}\,du = u^{\frac{1}{2}}\Big|_1^9 = (9)^{\frac{1}{2}} - (1)^{\frac{1}{2}}$$
$$= \boxed{2}$$

③ $$\int \frac{\boxed{\sin(x)\,dx}}{\cos^3(x)} \longrightarrow -du$$

$u = \cos(x) \longrightarrow du = -\sin(x)\,dx$

$$-\int \frac{du}{u^3} = -\int u^{-3}\,du = \frac{u^{-2}}{2} + C = \boxed{\frac{1}{2}(\cos x)^{-2} + C}$$

④ $$\int \frac{(\ln x)^2}{x}\,dx \qquad \boxed{\frac{dx}{x}} \longrightarrow du$$

$u = \ln x \longrightarrow du = \frac{1}{x}\,dx$

$$\int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{(\ln x)^3}{3} + C}$$

⑤ $$\int_0^{\pi/2} e^{\sin(x)}\,\boxed{\cos(x)\,dx} \longrightarrow du$$

Let $u = \sin(x) \longrightarrow du = \cos(x)\,dx$

$$\int e^u\,du = e^u \longrightarrow e^{\sin x}\Big|_0^{\pi/2} = e^{\sin(\frac{\pi}{2})} - e^{\sin(0)}$$
$$= e^1 - e^0 = \boxed{e-1}$$

6) $$\int_0^{\ln(5)} \frac{\boxed{e^x \, dx}}{1+e^{2x}} \xrightarrow{} du$$

$x = 0 : u = e^0 = 1$

$x = \ln(5) : u = e^{\ln(5)} = 5$

Let $u = e^x \longrightarrow du = e^x \, dx$

$$\int_1^5 \frac{1}{1+u^2}\, du = \arctan(u) \Big|_1^5 = \arctan(5) - \arctan(1)$$

$$= \boxed{\arctan(5) - \frac{\pi}{4}}$$