Lecture 3 - The Shell Method

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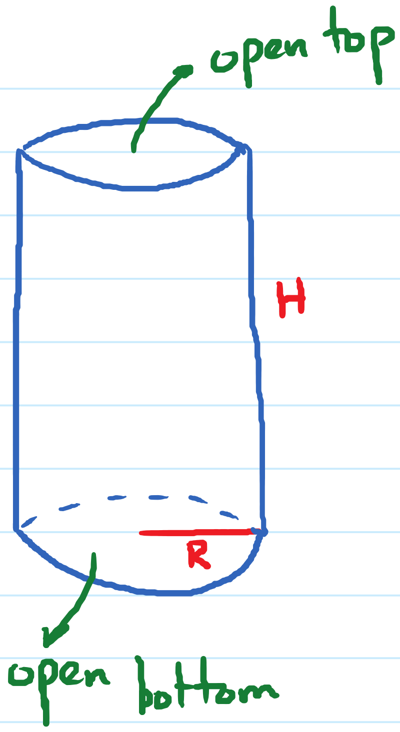
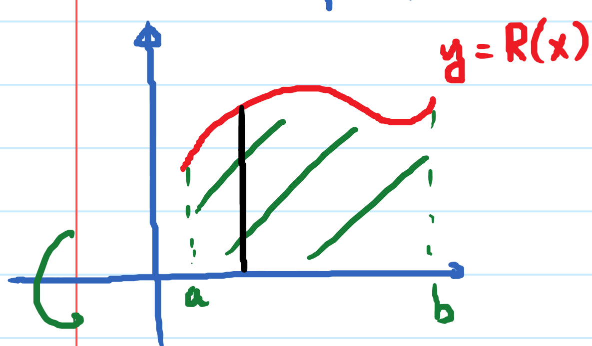
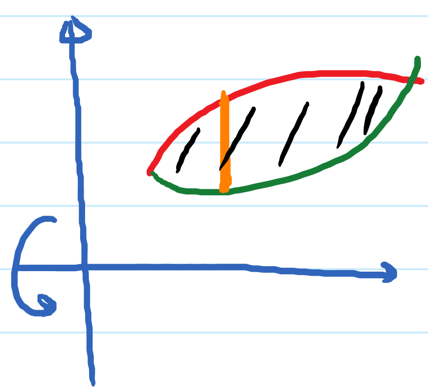
Question:



Surface area of this cylinder (shell) $= 2\pi R \cdot H$

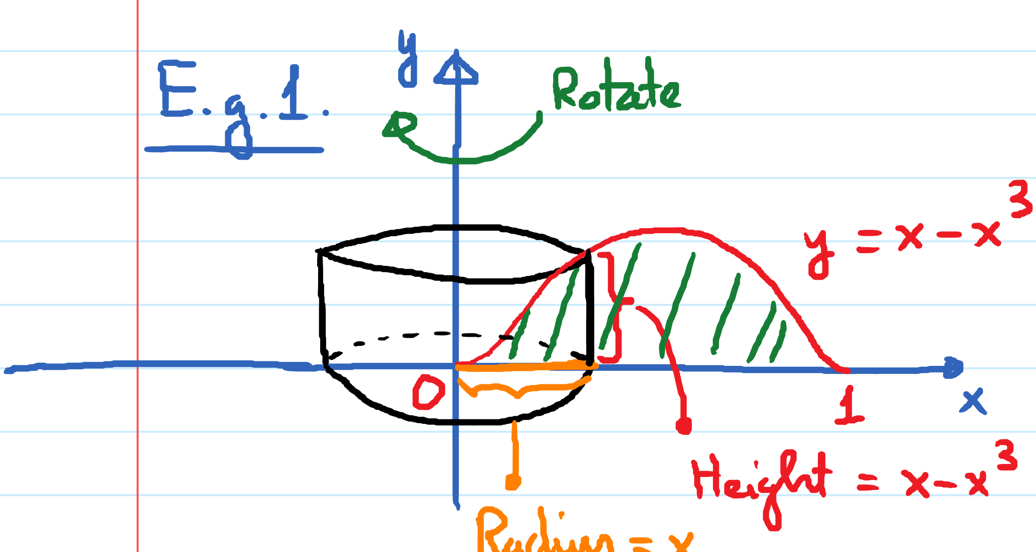
Note: For disk / washer method : cross sections are perpendicular to the axis of rotation.





$\longrightarrow$ For shell method, we will take the cross sections to be parallel to the axis of rotation.

E.g. 1.



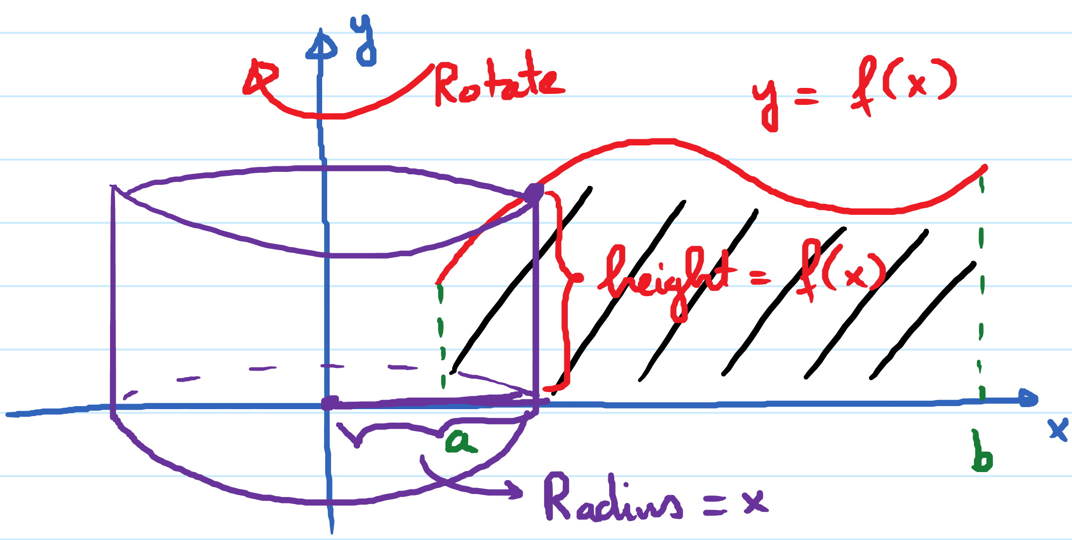
cross-section = cylinder (cylindrical shell) cross-section area = $2\pi(\text{Radius})(\text{Height})$

$$= 2\pi \cdot x \cdot (x - x^3)$$

$$V = \int_0^1 2\pi \cdot x\,(x - x^3)\,dx = 2\pi \cdot \int_0^1 (x^2 - x^4)\,dx$$

$$= 2\pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Bigg|_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{5}\right) = \boxed{\frac{4\pi}{15}}$$

Formulas for shell method



$$V = \int_a^b 2\pi \cdot (\text{Radius}) \cdot (\text{Height})\,dx$$

$$= 2\pi \int_a^b x \cdot f(x)\,dx.$$

If we rotate $x = g(y)$; $c \le y \le d$ about x-axis

$$V = 2\pi \cdot \int_c^d y\,g(y)\,dy.$$

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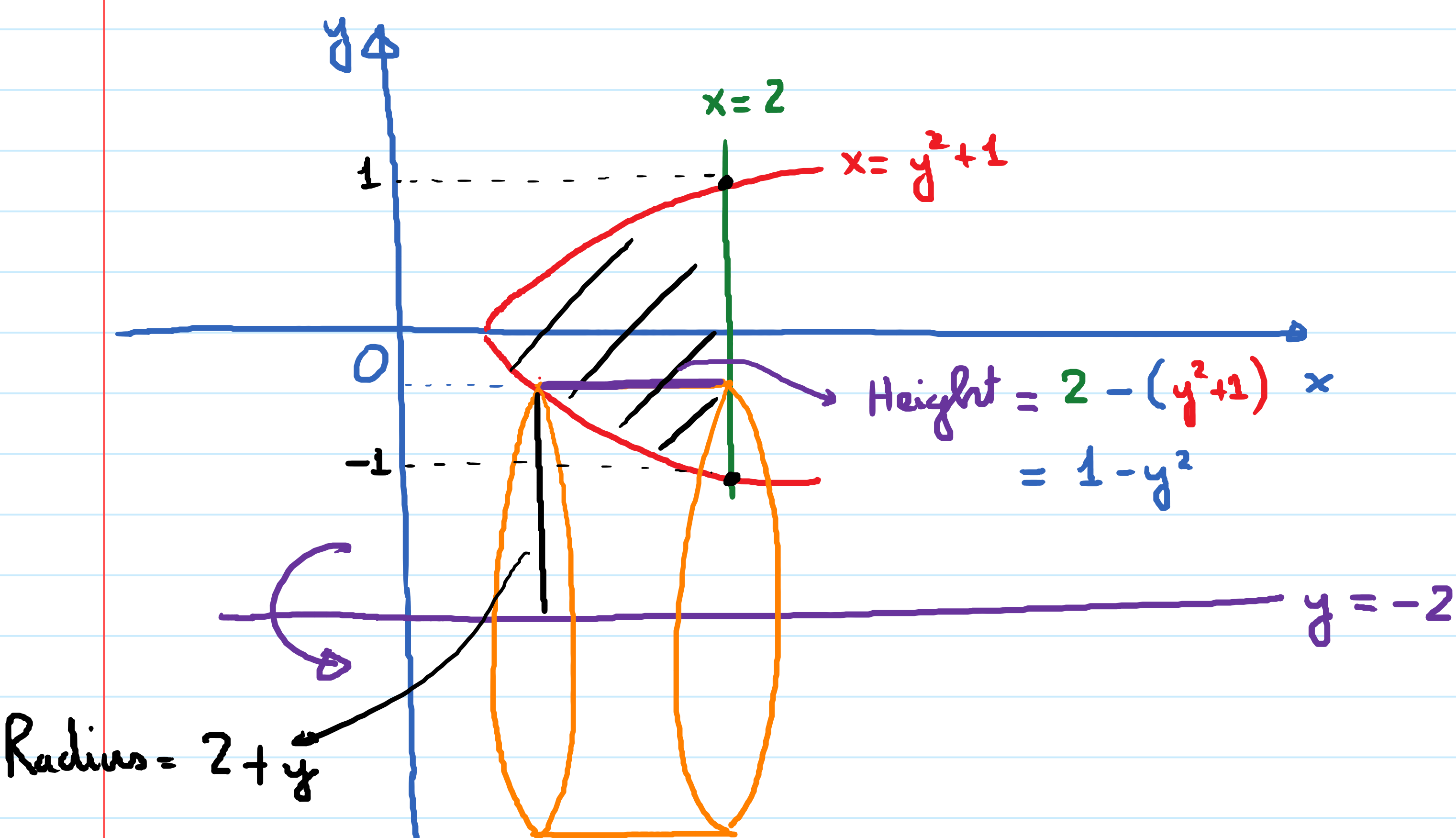
E.g. 2. Done on the board.

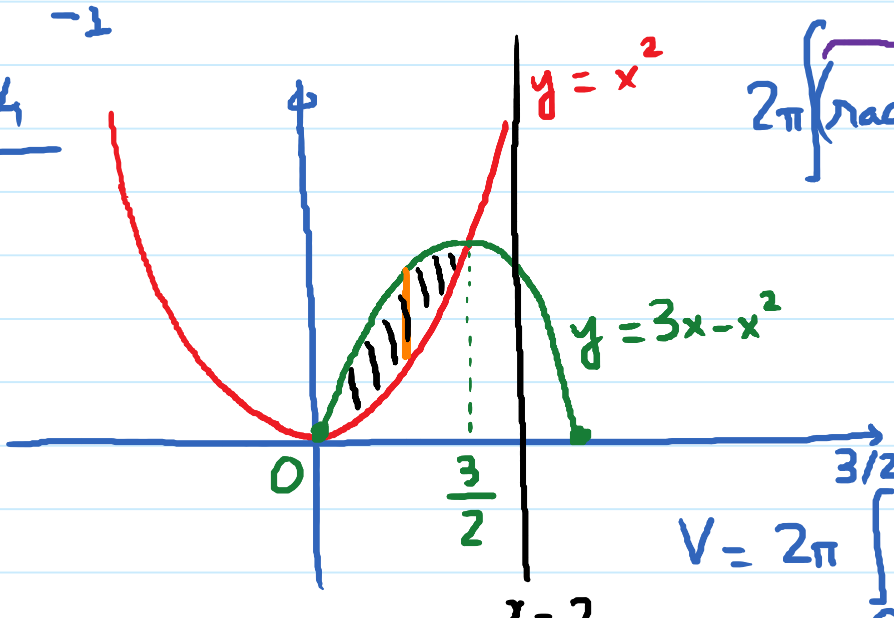
E.g. 3. Rotate about an axis that is neither x nor y.

$x = y^2 + 1$; $x = 2$. Rotate about $y = -2$



$$V = \int_{-1}^{1} 2\pi\,(2+y)\cdot(1-y^2)\,dy$$

E.g. 4



$$2\pi\int \overbrace{(\text{radius})}^{2-x}\underbrace{(\text{height})}\,dx$$

$$\boxed{\begin{aligned}&3x - x^2 - x^2\\ &= 3x - 2x^2\end{aligned}}$$

$$V = 2\pi\int_0^{3/2} (2-x)(3x-2x^2)\,dx$$