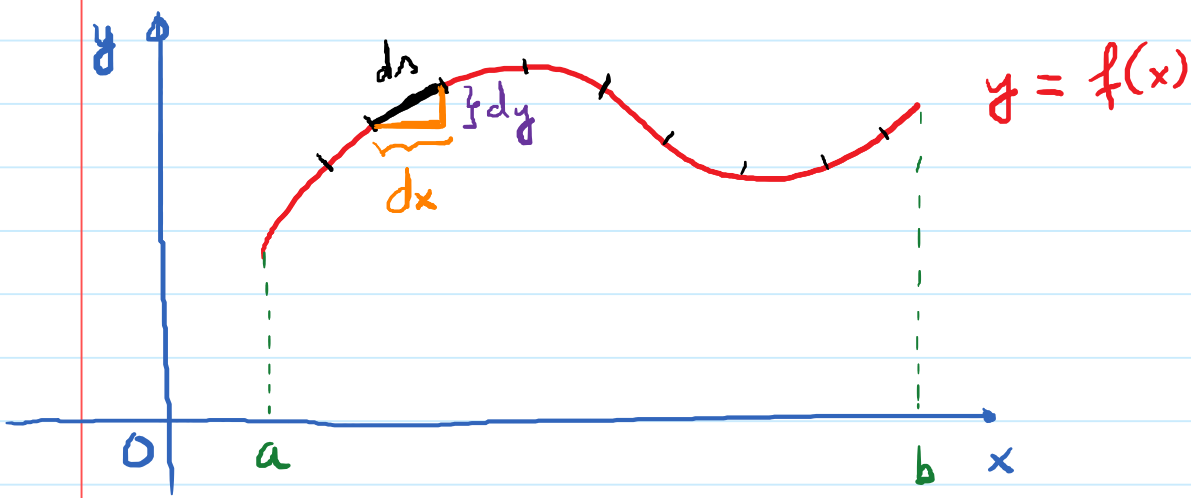
Lecture 4 - Arc length

Thursday, September 5, 2019 1:03 PM



L = length of the curve $y = f(x)$; $a \leq x \leq b$

Divide L into small segments, each segment has length ds

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$L = \int_a^b ds$$

To make this computable

$$ds = \sqrt{(dx)^2 \left[1 + \frac{(dy)^2}{(dx)^2}\right]}$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + (f'(x))^2}\, dx$$

Hence,

$$\boxed{L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx}$$

If the curve is given by $x = g(y)$; $c \leq y \leq d$

$$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$$

E.g.1.

$$L = \int_0^8 \sqrt{1 + \left(3x^{\frac{1}{2}}\right)^2}\, dx$$

$$= \int_0^8 \sqrt{1 + 9x}\, dx$$

$x = 0$: $u = 1$

$x = 8$: $u = 73$

let $u = 1 + 9x$. $du = 9\,dx \rightarrow dx = \frac{du}{9}$

$$L = \int_1^{73} \sqrt{u}\, \frac{du}{9} = \frac{1}{9}\int_1^{73} u^{\frac{1}{2}}\, du$$

$$= \frac{1}{9} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}} \Big|_1^{73}$$

$$= \frac{2}{27}\left((73)^{\frac{3}{2}} - 1\right)$$

E.g2. Work on the board

E.g.3 Work on the board