Lecture 2 - Volume by disk and washer method

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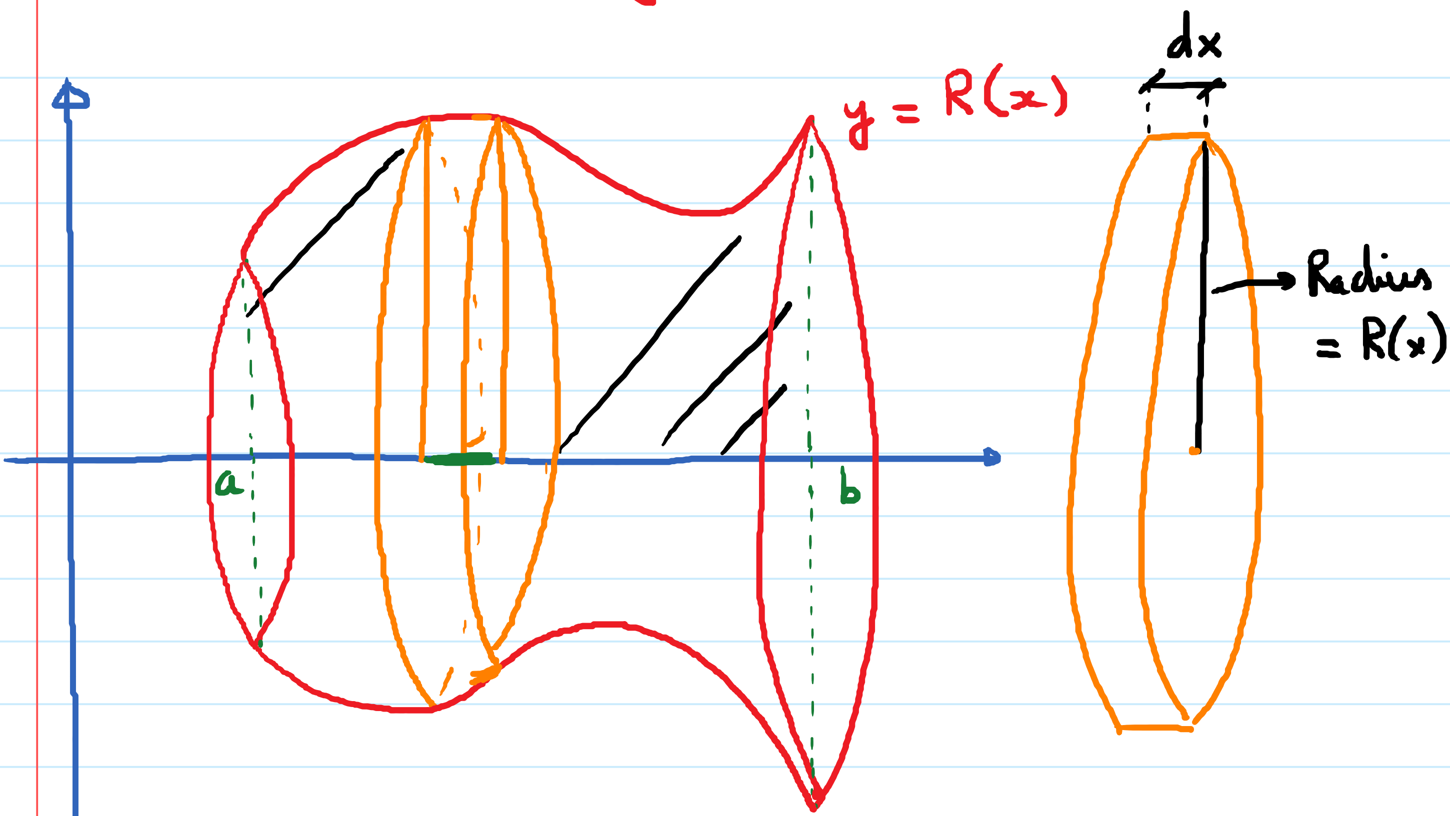
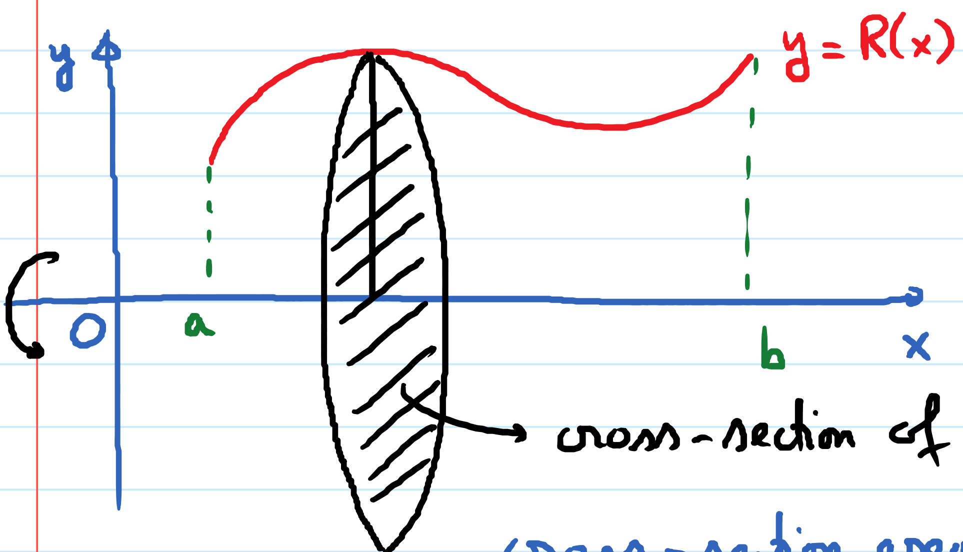


Volume of a small cylinder = (Base area) · height

$$= \pi \cdot [R(x)]^2 \cdot dx$$

$$V_{object} = \int_a^b \pi \cdot [R(x)]^2 \cdot dx$$

$$\boxed{V = \pi \cdot \int_a^b [R(x)]^2 \, dx.}$$



cross-section of object

cross-section area $= \pi \cdot (R(x))^2$

$$V = \int (\text{cross section area}) \cdot (\text{thickness})$$

In this case : cross section = disk.

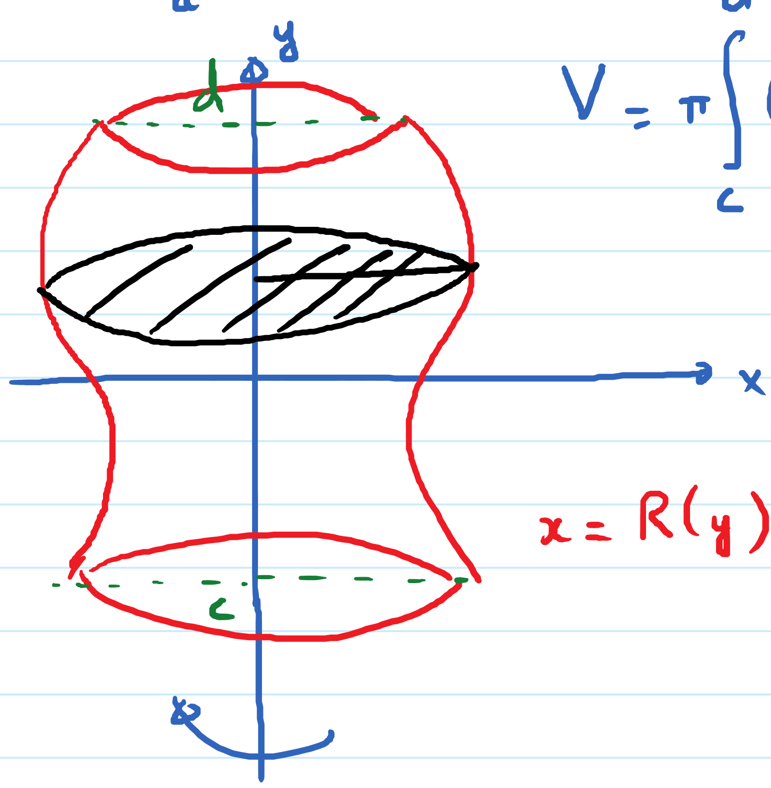
$\longrightarrow$ cross section area $= \pi \cdot (\text{Radius})^2$

$$V = \int \pi \cdot (\text{Radius})^2 \cdot (\text{thickness})$$

Radius $= R(x)$; thickness $= dx$

$$\longrightarrow V = \pi \int_a^b [R(x)]^2 \, dx$$

$$V = \pi \int_c^d (R(y))^2 \, dy$$



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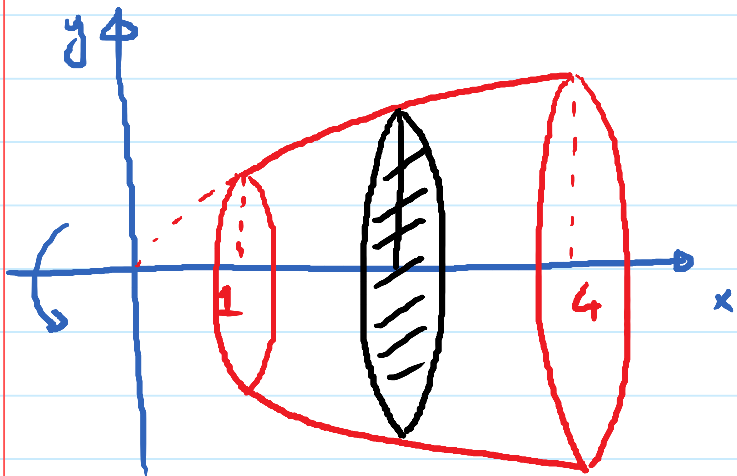
E.g. $y = \sqrt{x}$; $1 \leq x \leq 4$

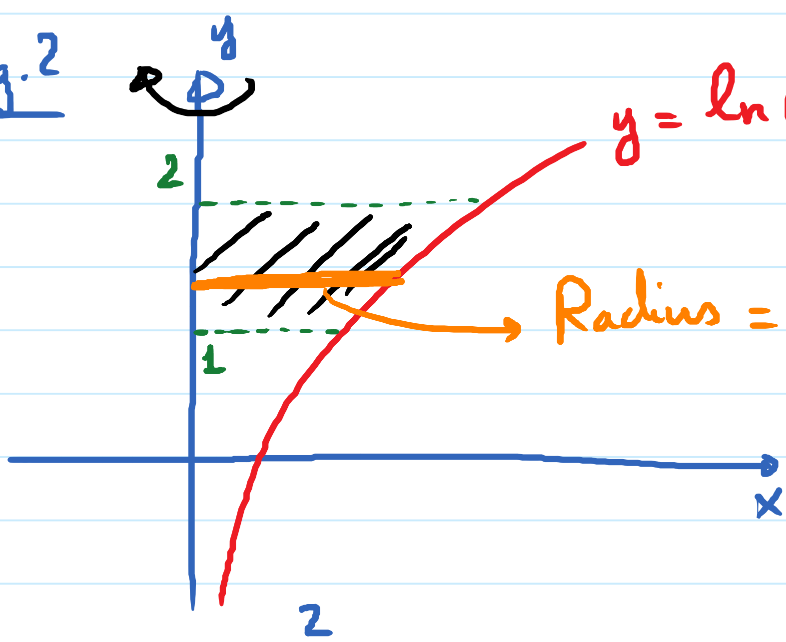


Radius $= \sqrt{x}$

$$V = \pi \cdot \int_1^4 \left(\sqrt{x}\right)^2 dx = \pi \cdot \int_1^4 x\,dx = \pi \cdot \frac{x^2}{2}\Big|_1^4$$

$$= \frac{15\pi}{2}$$

E.g. 2



$y = \ln(x) \rightarrow x = e^y$

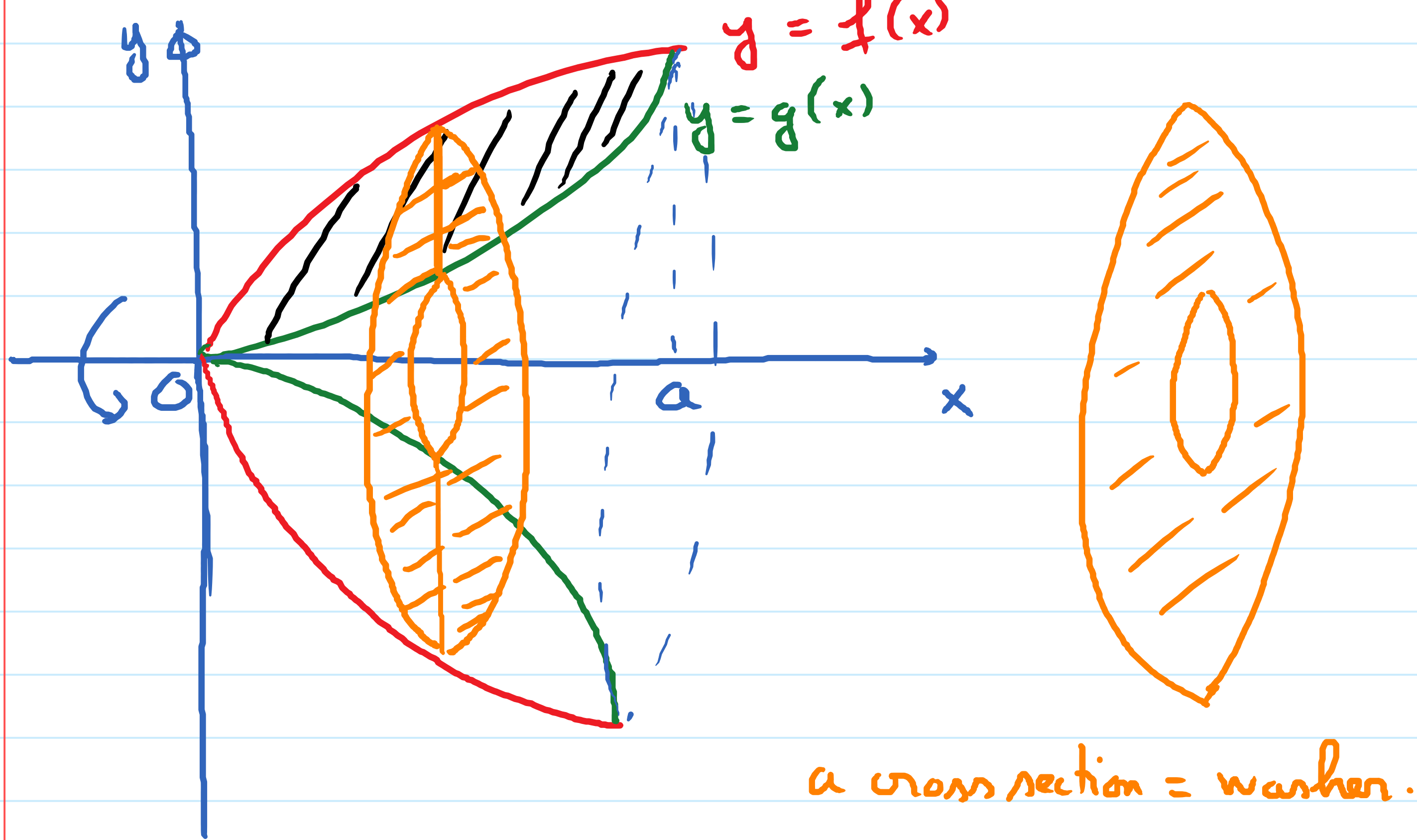
Radius $= e^y$

$$V = \pi \cdot \int_1^2 \left(e^y\right)^2 dy$$

$$= \pi \cdot \int_1^2 e^{2y}\,dy = \pi \cdot \frac{1}{2} e^{2y}\Big|_1^2$$

$$= \boxed{\frac{\pi}{2}\left(e^4 - e^2\right)}$$

Note: $V = \int (\text{cross section area}) \cdot (\text{thickness})$



$V = ?$

Area of washer $= A_{\text{larger disk}} - A_{\text{smaller disk}}$

$= \pi \cdot (f(x))^2 - \pi \cdot (g(x))^2$

cross section area $= \pi \cdot \left[\underbrace{(f(x))^2}_{\text{outer radius}} - \underbrace{(g(x))^2}_{\text{inner radius}} \right]$

$$V = \pi \cdot \int_0^a \left[[f(x)]^2 - [g(x)]^2 \right] dx$$

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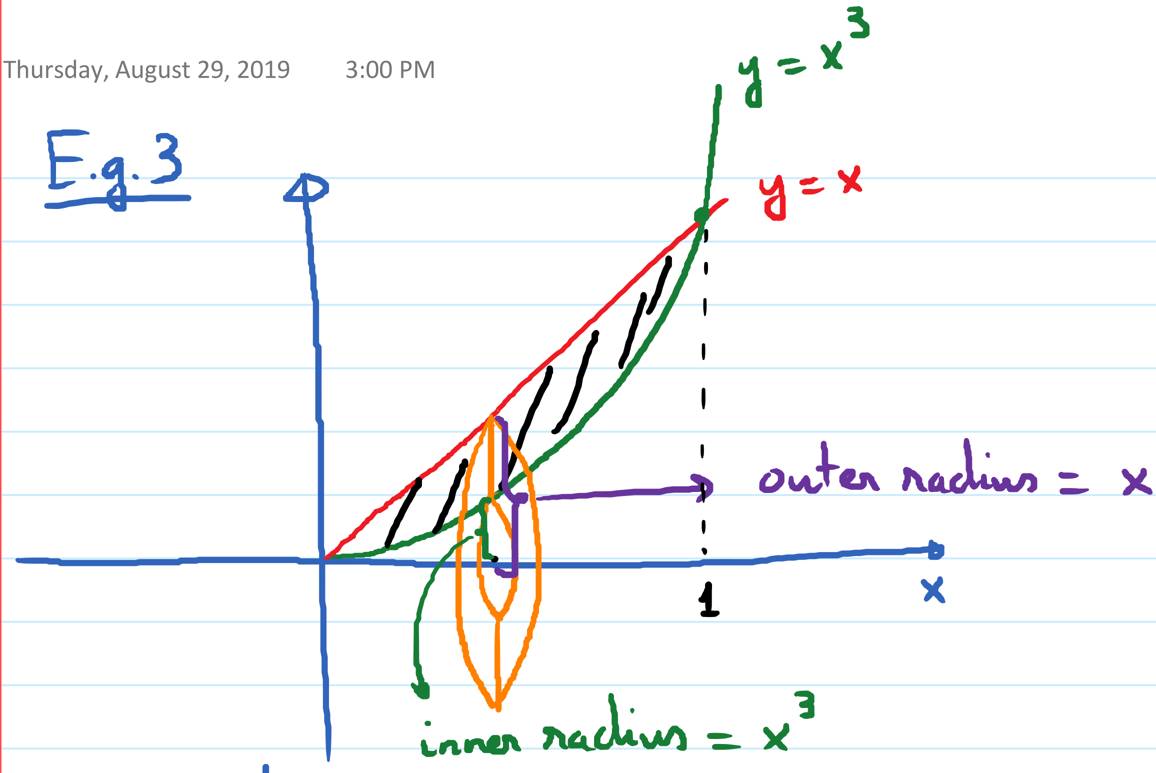
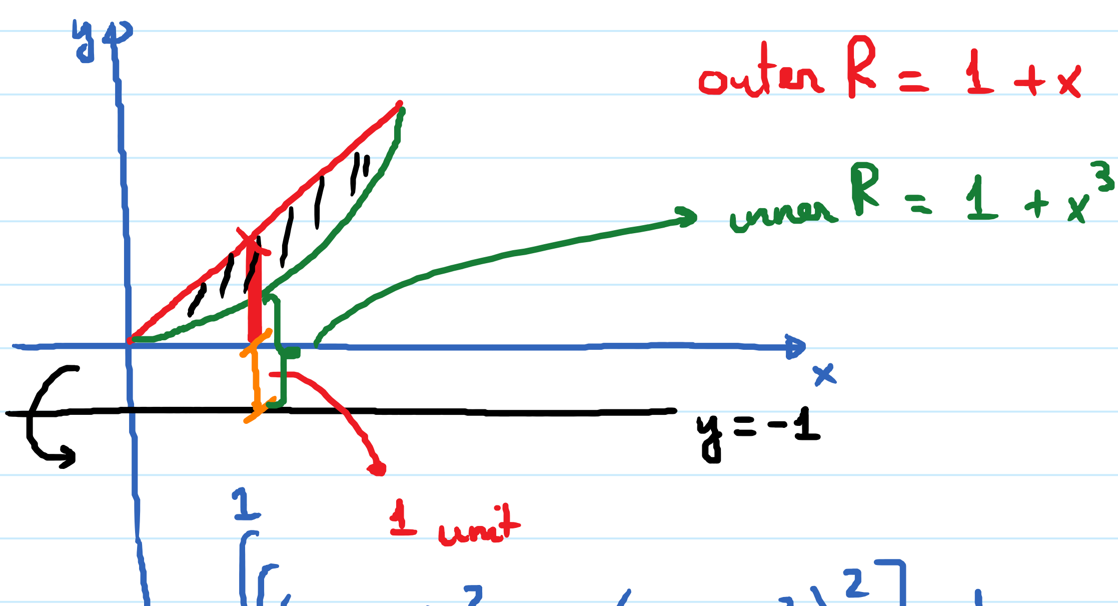
E.g.3



$$V = \pi \cdot \int_0^1 \left[x^2 - x^6\right] dx$$

$$= \pi \cdot \left(\frac{x^3}{3} - \frac{x^7}{7} \right) \Big|_0^1$$

$$= \pi \cdot \left(\frac{1}{3} - \frac{1}{7} \right) = \boxed{\frac{4\pi}{21}}.$$



$$\pi \int_0^1 \left[(1+x)^2 - (1+x^3)^2 \right] dx$$

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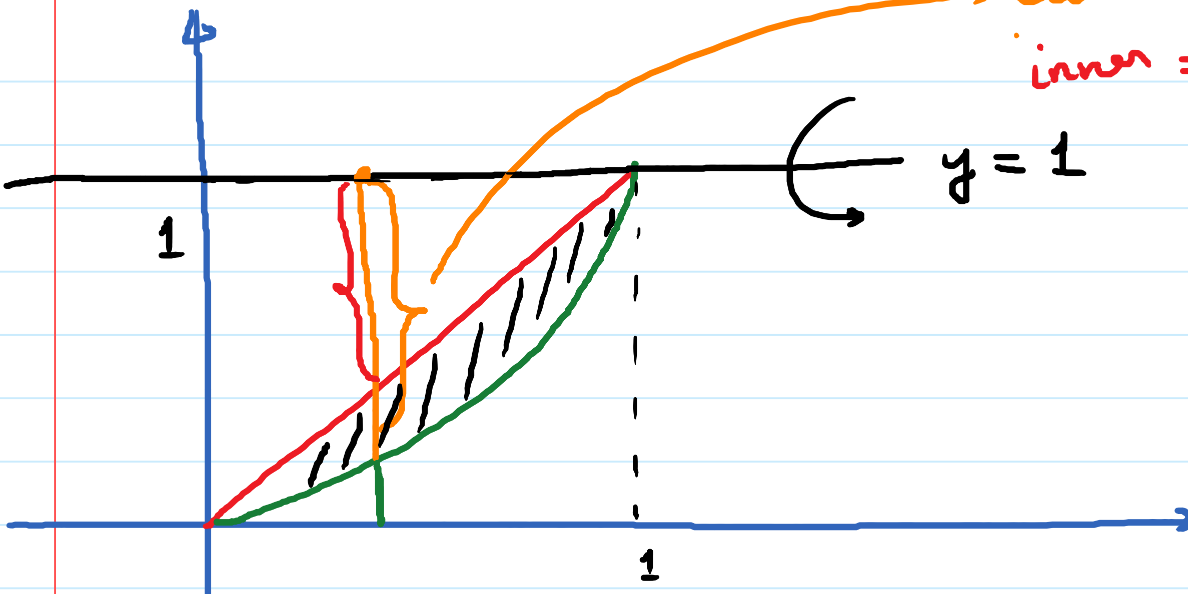
outer $= 1 - x^3$

inner $= 1 - x$



$$\pi \cdot \left[\left(1 - x^3\right)^2 - \left(1 - x\right)^2 \right]$$