# Lecture 2 ( Disk and Washer Method)

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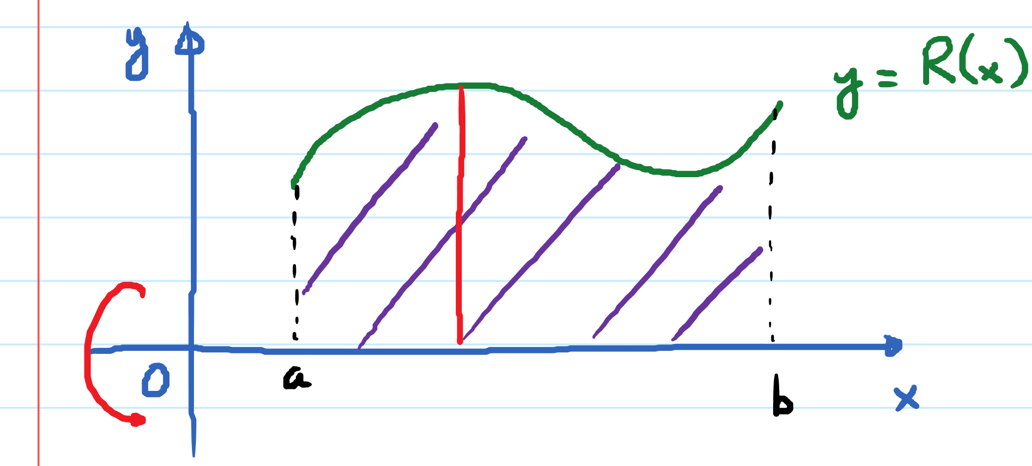
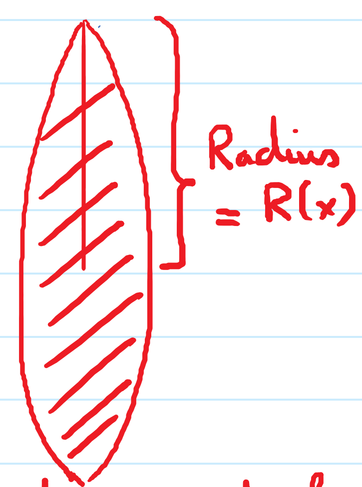
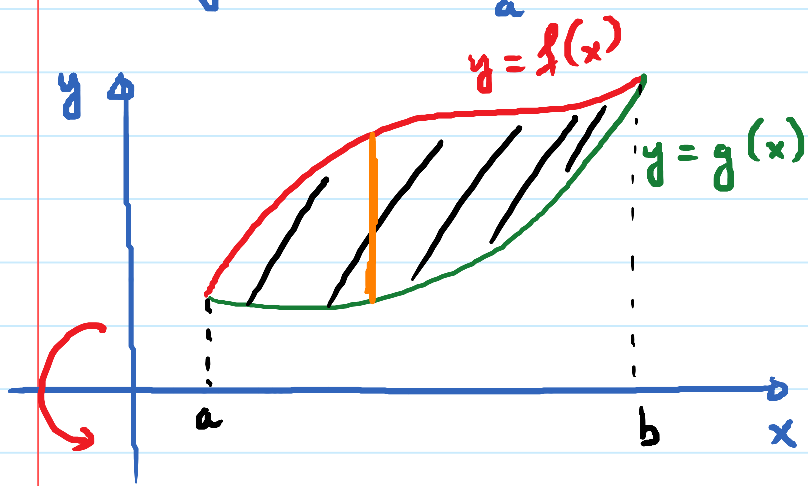
## Recall:

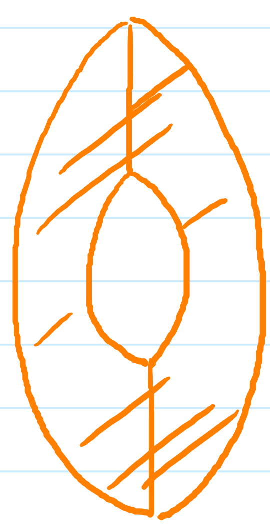



cross section = a disk

$$V_{object} = \pi \cdot \int_a^b [R(x)]^2 \, dx$$



Cross section = washer

$$V = \pi \int_a^b \Big(\underbrace{\text{outer radius}}_{f(x)}\Big)^2 - \Big(\underbrace{\text{inner radius}}_{g(x)}\Big)^2$$

$$= \pi \int_a^b \Big( [f(x)]^2 - [g(x)]^2 \Big) \, dx$$

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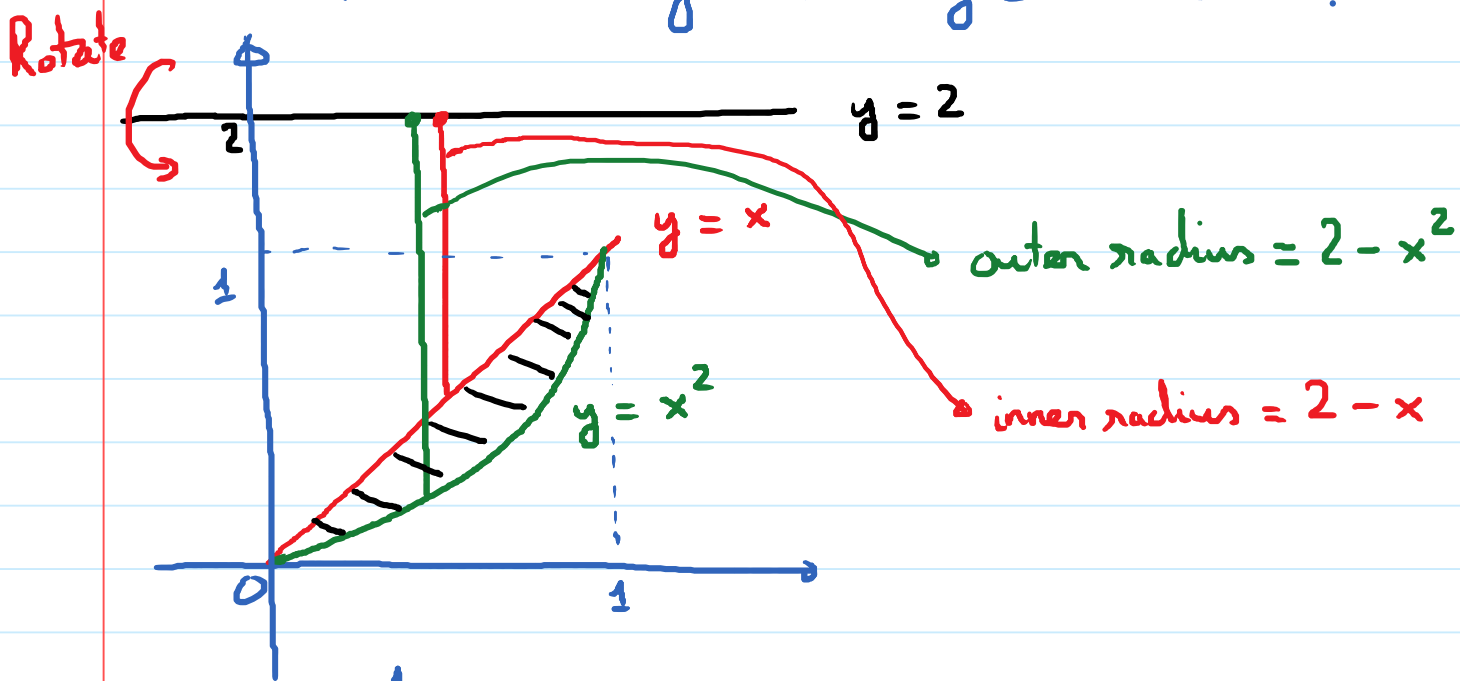
Now, rotate about an axis other than x or y.

E.g. 4 $y = x$ ; $y = x^2$.

Rotate the region about $y = 2$. Find V.




$$V = \pi \int_0^1 \left[ (\underbrace{2 - x^2}_{\text{out. R.}})^2 - (\underbrace{2 - x}_{\text{inner. R}})^2 \right] dx$$

$$= \pi \cdot \int_0^1 \left(4 - 4x^2 + x^4 - 4 + 4x - x^2\right) dx$$

$$= \pi \cdot \int_0^1 \left(x^4 - 5x^2 + 4x\right) dx$$

$$= \pi \cdot \left(\frac{x^5}{5} - 5\frac{x^3}{3} + 2x^2\right) \Bigg|_0^1$$

$$= \pi \cdot \left(\frac{1}{5} - \frac{5}{3} + 2\right) = \boxed{\frac{8\pi}{15}}.$$

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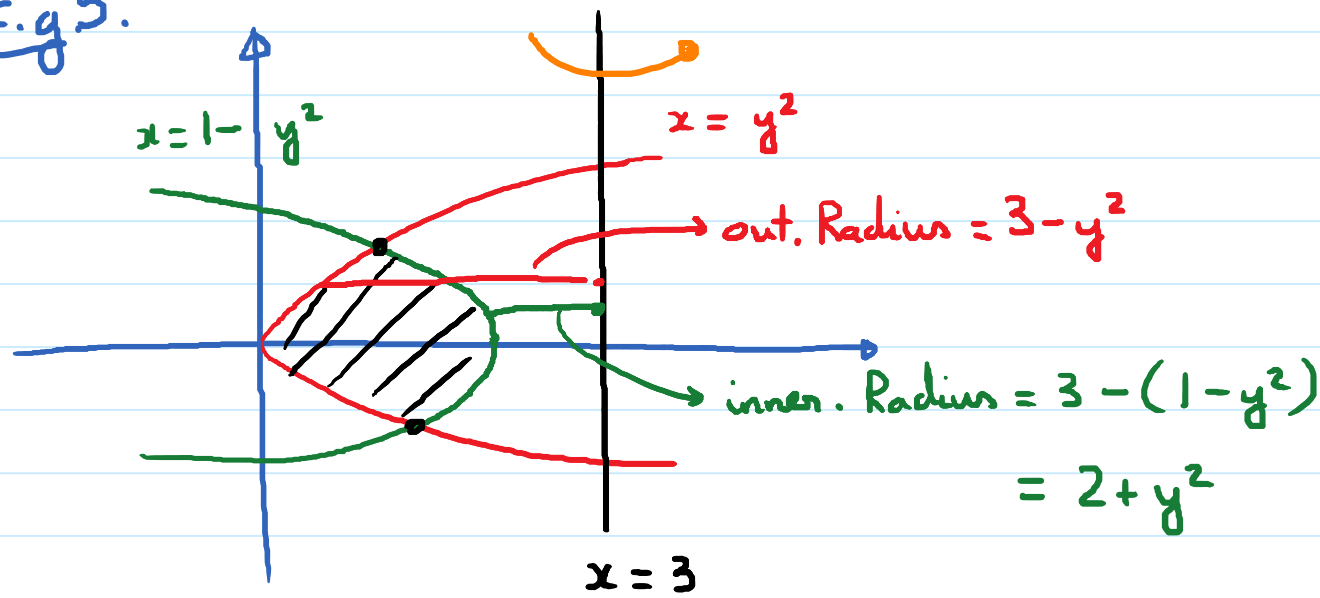
E.g 5.




Points of intersection: $1 - y^2 = y^2 \rightarrow y^2 = \frac{1}{2} \rightarrow y = \pm \frac{\sqrt{2}}{2}$

$$\boxed{V = \pi \cdot \int_{-\sqrt{2}/2}^{\sqrt{2}/2} \left[ (3 - y^2)^2 - (2 + y^2)^2 \right] dy.}$$

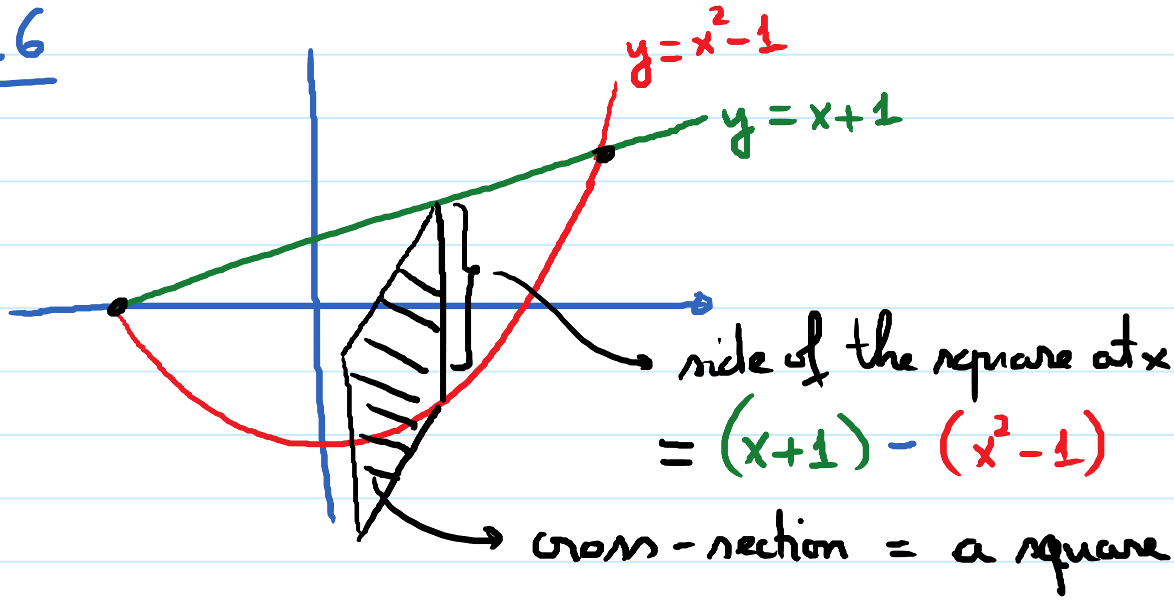
→ Main idea: To find the volume of an object whose cross section area is known we find

$$\int_{\text{lower bound}}^{\text{upper bound}} (\text{cross section area})(\text{thickness})$$

If cross sections are perpendicular to x-axis (y) and cross section area at x (y) is given by the formula $A(x)$, ($A(y)$) then

$$V = \int_a^b A(x)\,dx \quad (A(y))\,dy$$

## E.g. 6




Cross-section area formula

$$A(x) = \text{area of a square} = (\text{side})^2$$

$$\text{side} = (x+1) - (x^2-1) = -x^2+x+2$$

So,

$$A(x) = \left(-x^2+x+2\right)^2$$

Points of intersection:

$$x^2-1 = x+1 \rightarrow x^2-x-2=0$$

$$(x-2)(x+1)=0$$

$$x=2;\ x=-1$$

$$V = \int_{-1}^{2} A(x)\,dx = \int_{-1}^{2} \left(-x^2+x+2\right)^2 dx = \boxed{8.1}$$