1.5. Quadratic Equations

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Objectives: (1) Definition of a quadratic equation.

(2) Solve Quadratic Equations by Factoring

(3) Solve Quadratic Equations by Square Root Property

(4) Solve Quadratic Equations by the Quadratic Formula

(1) Def. of a quadratic equation:

E.g. $x^2 - 7x + 10 = 0$

A quadratic equation is an equation of the form:

$$ax^2 + bx + c = 0$$

where a, b, c are real numbers and $a \neq 0$.

In the above example: $a = 1$; $b = -7$; $c = 10$

(2) Solve quadratic equations by factoring

E.g. Solve $x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$ (Factor)

By the zero product principle, we set each factor equal to zero:

$x - 5 = 0$ or $x - 2 = 0$

$\boxed{x = 5}$ $\boxed{x = 2}$

Solution set: $\boxed{\{5, 2\}}$

E.g. Solve: $4x^2 - 2x = 0$

$2x(2x - 1) = 0$ (Factor)

Set each factor equal to zero:

$2x = 0$ or $2x - 1 = 0$

$\boxed{x = 0}$ $2x = 1 \rightarrow \boxed{x = \frac{1}{2}}$

Solution set: $\boxed{\{0, \frac{1}{2}\}}$

E.g. $2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$ (Subtract 4 from both sides. One side must be zero when we solve by factoring)

$(2x - 1)(x + 4) = 0$ (Factor)

Set each factor equal to 0

$2x - 1 = 0$ or $x + 4 = 0$

$\boxed{x = \frac{1}{2}}$ $\boxed{x = -4}$

Solution set: $\boxed{\left\{\frac{1}{2}, -4\right\}}$

E.g. $2x^2 + x = 1$

$2x^2 + x - 1 = 0$ (Right side = 0)

$(2x - 1)(x + 1) = 0$ (Factor)

Set each factor equal to 0:

$2x - 1 = 0$ or $x + 1 = 0$

$x = \frac{1}{2}$, $x = -1$

Solution set: $\boxed{\left\{\frac{1}{2}, -1\right\}}$

③ Solve quadratic equations by the square root property

take square root of both sides

E.g. $x^2 = 4 \longrightarrow x = \pm\sqrt{4} = \pm 2$

In general, the square root property says that if u is an expression and d is a number and we have: $u^2 = d$

Then: $u = \sqrt{d}$ or $u = -\sqrt{d}$

We can write this in an equivalent way:

$$u^2 = d \longrightarrow u = \pm\sqrt{d}$$

E.g. Solve quadratic equations by the square root property:

E.g.ⓐ: $3x^2 - 15 = 0$

Isolate x^2: $3x^2 = 15$ (Add 15 to both sides)

$x^2 = 5$ (Divide both sides by 3)

By the square root property, we take square root of both sides:

$$x = \pm\sqrt{5}$$

Solution set: $\boxed{\{\sqrt{5}, -\sqrt{5}\}}$

Note: Before you can apply the Square Root Property, a squared expression must be isolated on one side of the equation.

E.g. (b) $5x^2 - 45 = 0$

$$\longrightarrow 5x^2 = 45 \longrightarrow x^2 = 9$$

$$\longrightarrow x = \pm\sqrt{9} = \pm 3$$

Solution set: $\boxed{\{3, -3\}}$

E.g. (c) $(x-1)^2 - 81 = 0$

$$(x-1)^2 = 81$$

By the square root property:

$$x - 1 = \pm\sqrt{81} = \pm 9$$

$x - 1 = 9$ or $x - 1 = -9$

$x = 10$; $x = -8$

Solution set: $\{10, -8\}$

E.g. (d) $(2x + 3)^2 - 7 = 0$

$(2x + 3)^2 = 7$

By Square Root Property:

$2x + 3 = \pm\sqrt{7}$

$2x = -3 \pm \sqrt{7}$

$x = \frac{-3 \pm \sqrt{7}}{2}$

Solution set: $\left\{\frac{-3+\sqrt{7}}{2}, \frac{-3-\sqrt{7}}{2}\right\}$

(4) Solve Quadratic equations by using the Quadratic Formula.

The solutions of a quadratic equation

$$ax^2 + bx + c = 0 \; ; \; a \neq 0$$

are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is the quadratic formula.

E.g. Solve the equation

$$6x^2 - 5x - 4 = 0$$

$a = 6$; $b = -5$; $c = -4$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot (6) \cdot (-4)}}{2 \cdot (6)}$$

$$x = \frac{5 \pm \sqrt{121}}{12} = \frac{5 \pm 11}{12}$$

$$x = \frac{5 + 11}{12} \; ; \; x = \frac{5 - 11}{12}$$

$$x = \frac{16}{12} = \frac{4}{3} \; ; \; x = \frac{-6}{12} = -\frac{1}{2}$$

Solution set: $\left\{ \frac{4}{3} ; -\frac{1}{2} \right\}$

E.g. Solve: $2x^2 - 6x + 1 = 0$

$a = 2 \; ; \quad b = -6 \; ; \quad c = 1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot (2) \cdot (1)}}{2 \cdot (2)}$$

$$x = \frac{6 \pm \sqrt{28}}{4} = \frac{6 \pm \sqrt{4 \cdot 7}}{4}$$

Simplify

$$x = \frac{6 \pm \sqrt{4} \cdot \sqrt{7}}{4} = \frac{6 \pm 2\sqrt{7}}{4}$$

$$x = \frac{\cancel{2}(3 \pm \sqrt{7})}{\cancel{4}\,2} = \frac{3 \pm \sqrt{7}}{2}$$

Solution set: $\left\{ \frac{3+\sqrt{7}}{2} ; \frac{3-\sqrt{7}}{2} \right\}$